

FPGA Based Adaptive Filter Design Using Least PTH-Norm Technique

Srishtee Chaudhary, Rajesh Mehra

Abstract—Adaptive filters are considered nonlinear systems; therefore their behavior analysis is more complicated than for fixed filters. As adaptive filters are self-designing filters, their design can be considered less involved than in the case of digital filters with fixed coefficients. This paper presents simulation of Low Pass FIR Adaptive filter using least mean square (LMS) algorithm and least Pth norm algorithm. LMS algorithm is a type of adaptive filter known as stochastic gradient-based algorithms as it utilizes the gradient vector of the filter tap weights to converge on the optimal wiener solution whereas Least Pth does not need to adapt the weighting function involved and no constraints are imposed during the course of optimization. In this paper FPGA implementation of a low pass FIR filter is done using least Pth-norm technique. The performance of both approaches is compared.

Index Terms— Adaptive filters, FIR , Least Pth norm, LMS, Matlab, FPGA.

I. INTRODUCTION

Adaptive filter is a filter that self-adjusts its transfer function according to an optimization algorithm driven by an error signal. Because of the complexity of the optimization algorithms, most adaptive filters are digital filters. An adaptive filter is required when either the fixed specifications are unknown or the specifications cannot be satisfied by time-invariant filters [1]. An adaptive filter is a nonlinear filter since its characteristics are dependent on the input signal. However, if we freeze the filter parameters at a given instant of time, than adaptive filters considered are linear in the sense that their output signals are linear functions of their input signals[2]. As the signal into the filter continues, the adaptive filter coefficients adjust themselves to achieve the desired result, such as identifying an unknown filter or canceling noise in the input signal. Adaptive filtering can be considered as a process in which the parameters used for the processing of signals changes according to some criterion. Adaptive filters are dynamic filters which iteratively alter their characteristics in order to achieve an optimal desired output. An adaptive filter algorithmically alters its parameters in order to minimize a function of the difference between the desired output and its actual output. To define the self-learning process, select the adaptive algorithm used to reduce the error between the output signal $y(k)$ and the desired signal $d(k)$ [3]. There are various algorithms involved for the filtering depending upon the applications and the requirements.

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To construct an adaptive filter it has to be considered that which method is to be used to update the coefficients of selected filter and whether to use an IIR filter or FIR filter. For designing an adaptive filter algorithm plays a vital role. The algorithm has to be practically implemented, has to adapt the coefficients quickly and provide the desired performance.

II. ADAPTIVE FILTERING ALGORITHMS

Adaptive filtering can be classified into three categories: adaptive filter structures, adaptive algorithms, and applications. The choice of algorithm is highly dependent on the signals of interest, the operating environment, as well as the convergence time required and computation power available. An adaptive digital filter can be built up using an IIR (Infinite impulse response) or FIR (Finite impulse response) filter. Adaptive FIR filter structure is most commonly used adaptive FIR filter structure and is the transversal filter which implements an all-zero filter with a canonic direct form (without any feedback). FIR is inherently stable because its structure involves forward paths only, no feedback exists. The presence of feed back to the input may lead the filter to be unstable and oscillation may occur. For this adaptive FIR filter structure, the output is a linear combination of the adaptive filter coefficients. Alternative adaptive FIR filter structures improve performance in terms of convergence speed [4]. For simple implementation and easy analysis; most adaptive IIR filter structures use the canonic direct form realization. Some other realizations are also presented to overcome some drawbacks of canonic direct form realization, like slow convergence rate and the need for stable monitoring.

An algorithm is a procedure used to adjust adaptive filter coefficients in order to minimize the cost function. The algorithm determines important features of adaptive procedure, such as computational complexity, convergence to suboptimal solutions, biased solutions, objective cost function and error signal. The algorithm used in equalization is LMS and is known for its simplification, low complexity and better performance in different running environments [5]. Further symmetric approach can be employed to reduce the complexity with partial serial MAC based approach to optimize speed and area [6]. Fractionally spaced equalizer (FSE) can be used to compensate for channel distortion before aliasing effects occur due to symbol rate sampling. FSE is used to reduce computational requirements and to improve convergence [7].

Further Fast Block Least Mean Square (FBLMS) is one of the fastest and computationally efficient adaptive algorithms. Distributed Arithmetic further enhances the throughput of FBLMS algorithm with reduced delay, minimum area requirement and reduced hardware multipliers.



Distributed arithmetic (DA) is a bit level rearrangement of a multiply accumulate to hide the multiplications [8]. But the reduced hardware complexity of higher order filters was at the expense of increased memory and adder requirement. And the technique is suitable for higher order filters. It is a powerful technique for reducing the size of a parallel hardware multiply-accumulate that is well suited to FPGA designs. DA is one of the efficient techniques, in which, by means of a bit level rearrangement of a multiply accumulate terms; FFT can be implemented without multiplier.

The unconstrained optimization problem of Non-recursive filter to minimize the difference between actual and desired response of magnitude is solved using least squares design method for L2p norm [9]. Least square error design method for the optimal design of FIR filter showed that as the order of the filter is increased the ripple content in the stop band diminishes. Also the design using least Pth norm showed that the ripple content disappears and smoothen the response and give a constant response in stop band. The least Pth norm method doesn't need to update weighting functions, no constraints are imposed and design can start anywhere in parameter space [10]. Mixed-norm digital filter design methods provide the capability to design filters that have minimum deviation in the pass bands (using the L ∞ norm) and minimum broadband noise power in the stop-bands (using the L $_2$ norm). Filters that tradeoff between these two extremes (e.g., L $_4$ norm) are also possible [11]. The method allows for the rapid design of mixed-norm FIR filters by using an unconstrained optimization method.

Adaptive filtering has been successful in many applications, such as echo cancellation, noise cancellation, signal detection, system identification, channel equalization, and control. Some useful information about adaptive filtering application appears in the literature. The model of an adaptive filter using Least Mean Square Algorithm has been designed and simulated using Simulink and System Generator block synthesized with Xilinx Synthesis tool [15]. The proposed design consumed fewer resources, with maximum frequency 82.469MHz, Power 1.0139 W, Enhanced speed and Area Efficiency. Rafid Ahmed Khalil examines LMS algorithm of adaptive filtering and its application in acoustic echo cancellation system by employing a discrete signal processing in MATLAB for simulation with real acoustic signals. Also a hardware implementation of an adaptive filter have been developed using XC3S500E Xilinx FPGA chip, and VHDL language on RTL abstraction level [16]. The transversal FIR adaptive filters were used in only one of many methods of digital filtering.

An efficient filter structures with optimized code to create a system-on-chip (SOC) solution for various adaptive filtering problems specially unknown system identification.. Based on the error signal, the filter's coefficients are updated and becomes almost exactly as the unknown system' coefficients. Several different adaptive algorithms have been coded in VHDL as well as in MATLAB. The design is evaluated in terms of speed, hardware resources, and power consumption. System identification was mapped into a hardware description language, VHDL. The design was synthesized and implemented using FPGA (Xilinx Spartan3 3s200ft256 kit) [22] with 50 MHz clock. In general-purpose applications the hardware utilization and maximum frequency are two important designing criteria. The hardware implementation of adaptive least mean square (LMS) filter considering fixed

step size and self-correcting adaptive filter (SCAF) architectures on a spartan3 XC3S400 Field Programmable Gate Arrays (FPGA) chip has been considered. The performance of both approaches is compared in terms of convergence speed, hardware utilization and maximum frequency. The LMS filter is faster than the SCAF at the beginning, but it decreases gradually. The SCAF that has almost that hardware utilization is nearly same but maximum frequency is improved by SCAF [17].

III. LEAST SQUARE AND LEAST Pth NORM

When designing systems, it is important to have a systematic approach so that the design can be done timely and efficiently, which ultimately leads to lower cost. Among different algorithms for updating coefficients of an adaptive filter, LMS algorithm is used more because of its low computational processing tasks and high robustness. This algorithm is a member of stochastic gradient algorithm. It uses Mean Square Error (MSE) as a criterion. LMS uses a step size parameter, input signal and the difference of desired signal and filter output signal to frequently calculate the update of the filter coefficients set. The convergence time in case of LMS depends upon the step size parameter. If step size is small it will take long convergence time and smaller MSE. On the other hand large step size results faster convergence but large MSE. But if it is too large it will never converge. Thus the choice of step size determines the performance characteristics of adaptive algorithm in terms of convergence rate and amount of steady-state mean square error (MSE). The performance of LMS is a tradeoff between step size and filter order. The performance is also a tradeoff between convergence rate and MSE. To eliminate the tradeoff between convergence rate and MSE, one would use a variable step-size [12]-[13]-[14].

The commonly used algorithm LMS provides low complexity and stability. Further the need of filter to minimize the difference between actual and desired response of magnitude is solved using least Pth design method. But for FIR filters to a target frequency response one can apply a rectangular window to the impulse response. However, the resulting ringing is usually not acceptable and is not an optimal choice. For matching non-noisy target frequency responses, Least Pth is considered. The Pth optimization as a design tool is not new. It was used quite successfully for the minimax design of IIR filters. The method does not need to update the weighting function, and it is an unconstrained convex minimization approach. The approach has advantages as filter quality, mathematical verification of the properties such as causality, stability, etc using the pole zero and magnitude plots. The Least Pth norm algorithm has a larger gradient driving it to converge faster when away from the optimum. However, the LMS will have more desirable characteristics in the neighborhood of the optimum. The Least Pth norm algorithm is defined by the following cost function:

$$J_n = E[en^n] \tag{1}$$

$$\text{Where the error } en = dn + wn - cTnxn, \tag{2}$$

dn is the desired value, cn is the filter coefficient of the adaptive filter (with c_{opt} is its optimal value), xn is the input vector and wn is the additive noise.

IV. SIMULATION RESULTS

The optimal design of FIR filter using least Pth norm is implemented under MATLAB and is compared with least square algorithm. The filters vary in terms of desired filter characteristics and consequently in the number of coefficients depending upon the order of the filter. Simulation results are presented for the case of ten coefficient filter and a twenty coefficient filter. Comparisons are made with the Least square and least Pth norm algorithms. Figure 1, 2 and 3 shows the simulated results for 10 coefficients Low pass FIR filter. Figure 4, 5 and 6 shows the simulated results for 20 coefficient Low pass FIR filter. Figure 1 and 4 shows magnitude response with the sample frequency of 48 KHz for 10 coefficient and 20 coefficient Low pass FIR filter, Figure 2 and 5 shows pole/zero plot specifying the stability aspect for 10 and 20 coefficient filters. The magnitude response shows that filter implemented using least Pth norm with (p=4) converges faster and the filter implemented using least mean square converges slow. As the value of p increases the ripples are smooth.

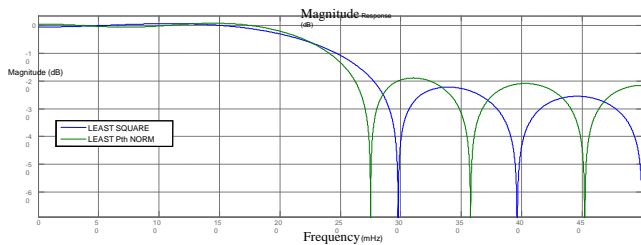


Figure 1 Magnitude Response (10 coefficient)

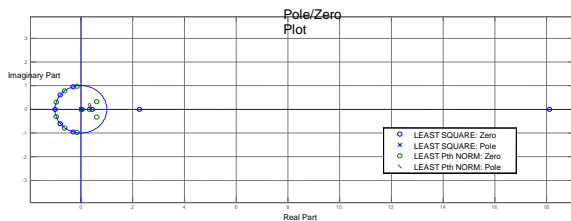


Figure 2 Pole and Zero Plot (10 coefficient)

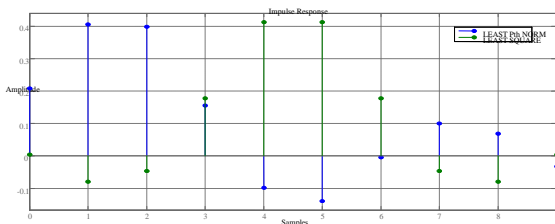


Figure 3 Impulse Response (10 coefficient)

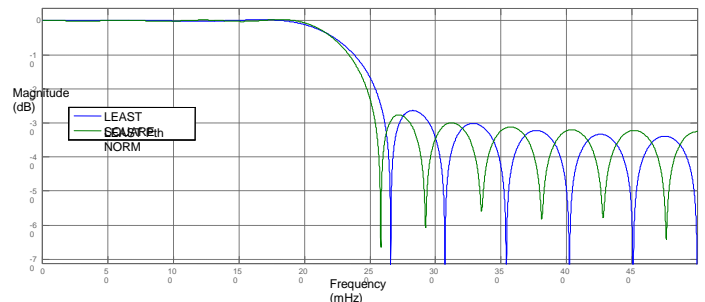


Figure 4 Magnitude Response (20 coefficient)

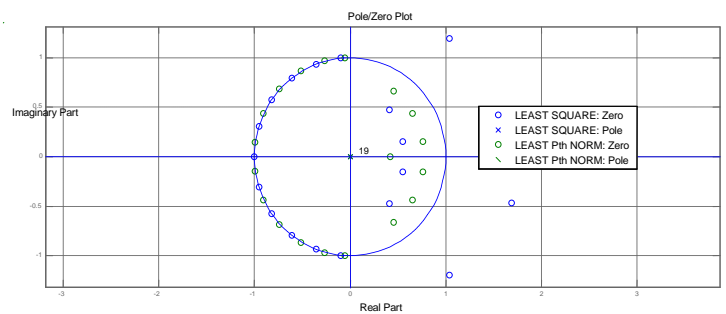


Figure 5 Pole and zero plot (20 coefficient)

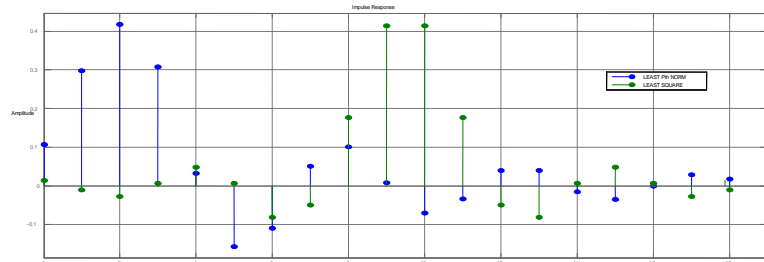


Figure 6 Impulse Response (20 coefficient)

Description	Least-square		Least Pth Norm	
	10 coeff	20 coeff	10 coeff	20 coeff
Filter Gain	.014	.004	.208	.107
Magnitude (at 10KHz)	-3.9	-2.4	-3.05	-1.8
Phase (at 10KHz)	-5.9	-12.4	-2.3	-3.3

Table: 1 Simulation results of Least Square and Least Pth

V. HARDWARE IMPLEMENTATION RESULTS

The developed VHDL code has been synthesized using Xilinx synthesizer tool (XST) and implemented on Spartan 3 XC35400 FPGA device. The resource utilization of Spartan 3 for 10 coefficient and 20 coefficient adaptive FIR filter using Least Pth norm is shown in the table.

Logic Utilization and speed	10 coeff filter	20 coeff filter	Available
No. of Slices	276	603	3584

No. of Slice Flip Flops	275	519	7168
No. of LUTs	370	854	7168
No. of multipliers	9	16	16
Maximum Frequency(MHz)	88.89	63.04	-
Minimum Period(ns)	11.124	15.862	-

Table 2: Resource Utilization & Speed Performance of FIR Filter (10 and 20 coefficients)

Finally the proposed design results based on Spartan 3 FPGA have been compared with [17].The proposed design has shown improvement in table 3 and 4 by consuming less number of slices available on target FPGA.

Logic Utilization and speed	LMS	LMS-SCAF	Least Pth
No. of Slices	783	759	276
No. of Slice Flip Flops	591	604	275
No. of LUTs	1304	1210	370
No. of multipliers	16	16	9
Maximum Frequency(MHz)	31.7	37.30	88.89

Table 3: Resource Utilization & Speed Performance of FIR Filter (10 coefficients)

Logic Utilization and speed	LMS	LMS-SCAF	Least Pth
No. of Slices	3261	3117	603
No. of Slice Flip Flops	935	814	519
No. of LUTs	5673	5593	854
No. of multipliers	16	16	16
Maximum Frequency(MHz)	15.115	22.4	63.04

Table 4 Resource Utilization & Speed Performance of FIR Filter (20 coefficients)

VI. CONCLUSION AND FUTURE SCOPE

The aim of the paper was to compare the performance parameters of Least square and Least Pth Norm algorithm for adaptive filtering. Here two Low pass FIR filters using 10 coefficients and 20 coefficients were simulated using MATLAB and then implemented on FPGA device. As the coefficient order increases both in Least square or Least Pth norm the multipliers, adders, multiplications per unit and additions per input sample increases. The Least Pth provided the better results. The least Pth provided better gain as compared to least square. The ripple content disappears in a similar way in both the cases but in case of least Pth norm ripples smoothen the response and give a constant response in stop band. Also the proposed design consumed considerably less resources in terms of slices to provide cost effective solutions.

Least Pth norm can be used to design optimal FIR filters using optimization algorithms for both linear phase and non-linear phase.

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