Geometry and Strength of a Shell of Velaroidal Type on Annulus Plan with Two Families of Sinusoids

Krivoshapko S. N., Gil-Oulbe MAthieu

Abstract- In this paper, the shells limited by two flat concentric circles are considered.Both families of coordinate lines are sinusoids. Their middle surfaces may be associated to the group of velaroidal type surfaces. Considered surfaces can find application in landscape architecture and also in design of some manufactured details and structures as consist of cyclically repeating identical elements.The stress-strain state of ashell outlined on the considered surface and loaded by the dead weight is defined.

Keywords – a velaroidal surface, a thin-walled shell, architecture, the stress-strain state.

Problem Statement. In our century of innovative ideas,structures and manufactured details, inconceivable from the point of view of the near past are created.Architects and mechanical engineers demand to create and to research new shapes and surfaces which can be described by the analytical equations for their application in various branches of science and techniques.

Statement of geometrical task. In this manuscript, new type of surfaces with two families of sinusoidal curvilinear coordinatesis offered for studying.The surface is limited by two flat concentric circles (Fig. 1).The specified signs show that this surface is not velaroidal surface, but, apparently, they may be associated to surfaces of velaroidal type.

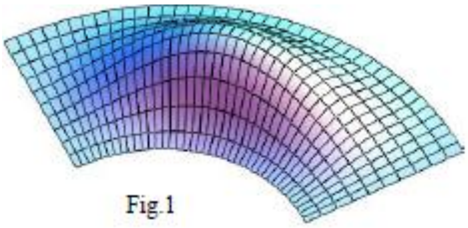

Fig.1

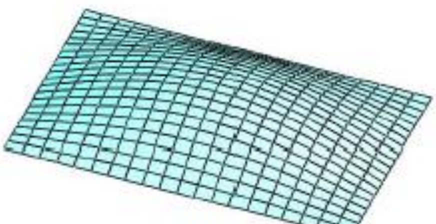

Fig. 2

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Velaroidal surface is a surface of translation on the flat rectangular plan witha generating curve of variable curvature [1].Thus, the surface is limited by four mutually orthogonal contour straight lines ($k_x = k_y = 0$) lying in the same plane. Up to date, three types of velaroidal surfaces are known. These are parabolic, elliptic and sinusoidal velaroids.

A sinusoidal velaroid generates by two families of half waves of the sinusoids lying in mutually perpendicular planes and facing by convexities into the same side (Fig. 2).Each set of sinusoids has the identical period. Sinusoidal velaroid is limited by a flat rectangular contour.

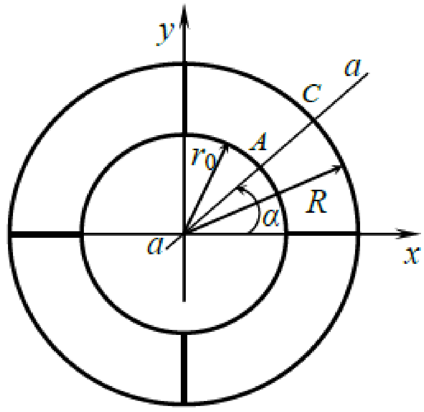

Fig. 3

The main material of geometrical research. Let's have two concentric circles with r_0 and R radiuses (Fig. 3) which we will take for contour curves of the surface. Let's assume that in any vertical section of a-a passing through the center O,a sinusoid lies, the $z = z(r) = -b\cos\frac{2\pi(r-c)}{R-r_0}+b,$

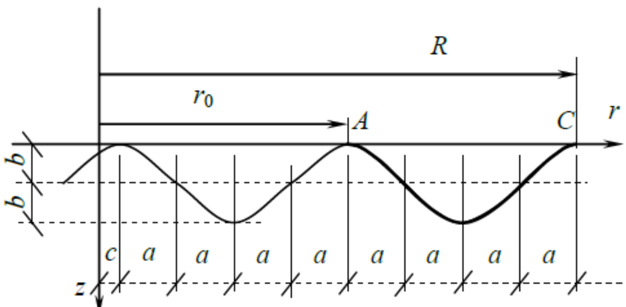

Fig. 4 $(a-a)$

wherebis the variable height of a half wave of the sinusoid, b = – 0.5B (cos(nα) – 1); B is the maximum height of a half wave; n is any number, $0 \le b \le B$.We will consider that part of the sinusoid being between points A and C ($r_0 \le r \le R$) is lying on a projected surface of velaroidal type.

The set of curvilinear coordinate lines in the peripheral direction, we will accept also in the form of sinusoids (Figure. 5)

$$z = z(\alpha) = -\tilde{n}(\cos(n\alpha) - 1),$$

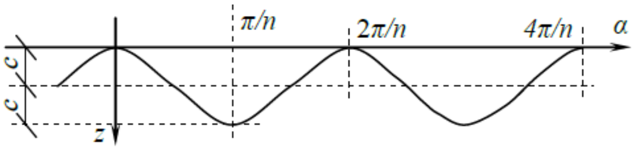

Fig. 5

wheren is the number of identical fragments of a surface in the district direction, accepted necessarily, but z =0for α = 0 always, i.e. line α = 0 on the surface is a straight line;

$$c = -B\left(\cos\frac{2\pi(r-c)}{R-r_0} - 1\right).$$

One fragment will be joined to the next similar fragment of the surface along the straight lines lying in the horizontal plane.For example, for Fig. 3, we have n = 4. Peripheral sinusoids on these straight lines have tangents which lie in the horizontal plane. Tangents to radial sinusoids in points of internal and external contours will lie in the horizontal planealso. Thus, we receive the parametrical equations of a projected surface as

$$x = x(r,\alpha) = r\cos\alpha,\ y = y(r,\alpha) = r\sin\alpha,$$

$$z = z(r,\alpha) = \frac{B}{2}(1-\cos(n\alpha))\left(1-\cos\frac{2\pi(r-c)}{R-r_0}\right), \quad (1)$$

$r_0 \le r \le R$, $0 \le \alpha \le 2\pi$, $a = (R-r_0)/4$, r = const arecurvilinear coordinate lines on the surface projected on the horizontal plane as concentric circles (Fig. 6).

According to the equations (1), we have that contour lines with z =0 will be if cos(nα) =1, i.e. at nα = 2mπ, or α_k = 2mπ/n is one cell of the surface, $0 \le \alpha \le \alpha_k$.

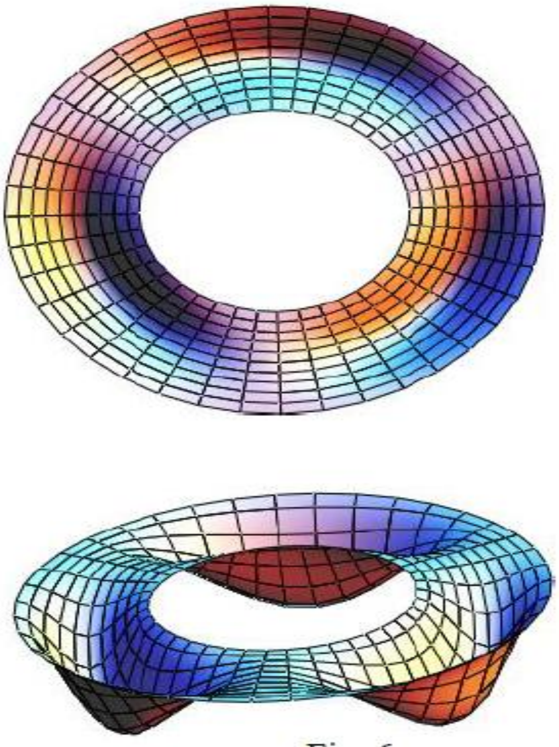

Fig. 6

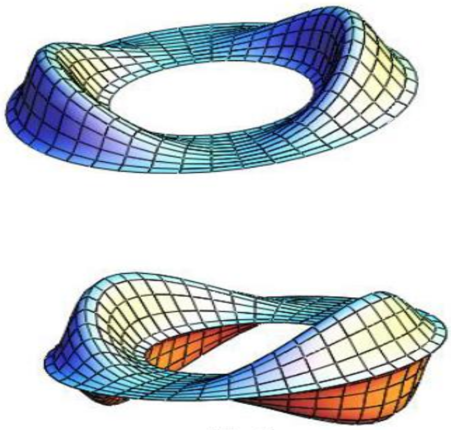

Fig. 7

The examined surface with n = 3, R = 4 m, r_0 = 2 m; B = 1 m, c = 0is shownin Fig. 6. In Fig. 7, the surface has n = 2. It is possible to create the closed surfacefrom two cavities of the same surface and contact will be carried out along two flat contour circles with radiuses r_0 and R and along the straight lines dividing two identical fragments of the surface. Interesting forms of surfaces of velaroidal type turn out if one will take r_0 = 0 (Fig. 4).The surface with r_0 = 0, R = 8 m; n= 8 and B = 0.5 m is presentedin Fig. 8. There is a special point in the central point of the surface.

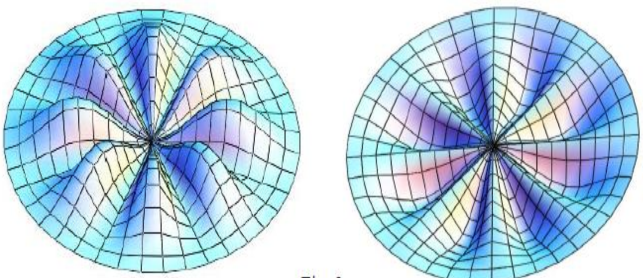

Fig. 8

The analysis of achievements and publications on velaroidal surfaces. Velaroidal surfaces in Russia unlike countries of Western Europe and America [1-3] don't enjoy wide popularity, except for the parabolic velaroid which form is transferred to covering of "Darbazi" [4, 5].This name occurred from the name of far historic Georgian covering.

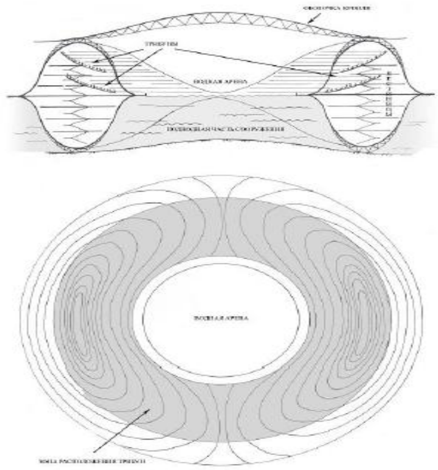

Fig. 9 [6]

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Following E. Torroja and F. Candela's manuals, it is necessary to have shown possibilities of thin-walled spatial shell structures and to increase interest to design of wide-span spatial structures taking into consideration the emergence of new materials such as fiber concrete and the fibrous reinforced polymeric composites, and taking in view the extending progress in numerical methods of analysis.

A group of students of Engineering Faculty of the Peoples' Friendship University of Russia training in architecture, became interested in results of geometrical researches of surfaces of velaroidal type. Within students' scientific society, they developed offers on application of the presented materials in landscape architecture of artificial and natural objects, in designing products for a decor and for shape generation of public buildings (Fig. 9).

Statement of task for calculation of a velaroidal shell. Apart from students in architecture, who made choice of these shells for master dissertations, designers became interested in velaroidal type shells. The velaroidal reinforced concrete shell was chosen as one of the variants of a covering of the dancing hall of «The Sports and Entertaining Center» (Fig. 10).

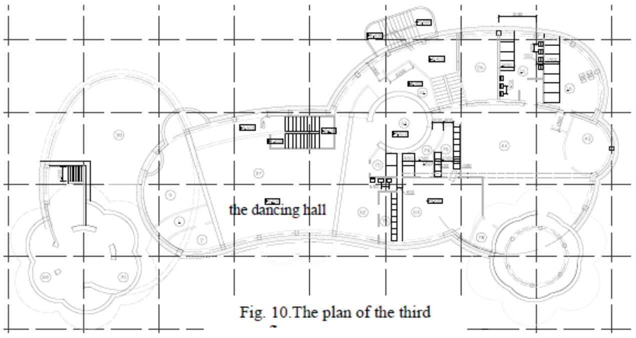
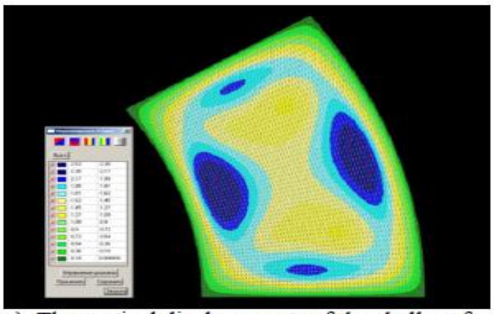


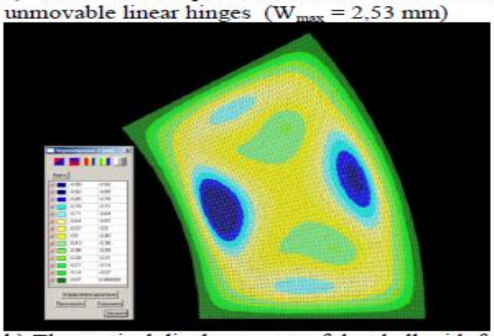
Fig. 10.The plan of the third

Geometrical parameters of the shell are the following:

r_0 = 20 m; R= 30 m; B= 1 m, $0 \le \alpha \le \alpha_k = 34^0$, therefore, c = 0; n= 2π/α_k = 17π/90 = 0.189π. Besides, it was taken for the shell h = 10 cm, E = 30000 MPa, Poisson's ratio ν = 0.17. The shell is loaded by the dead weight equal to 3 kPa.



a) The vertical displacements of the shell on four unmovable linear hinges (W_{max} = 2,53 mm)



b) The vertical displacements of the shell with four rigidly fixed edges (W_{max} = 0,99mm)

Fig. 11

The standard program complex Structural CAD (SCAD) was applied for static analysis. Results of calculation of the vertical displacements (W_z) are presented in Fig. 11. The authors have the results of calculation of the normal, the tangent, and the shearing forces, the bending and the twisting moments too reckoned per unit of curvilinear coordinates' length.

CONCLUSION

Between the 1920s and 1960s, considered the "golden age of thin shell structures" the thin-walled shell constructions were developed as an engineering solution in order to achieve large spans for industrial, commercial or public structures. After the 30 years interval, a new period of renascence of interest to shells begins [7]. In 2000s, new materials, shapes of shells and methods of their building were created[8] and the authors believe that the work would be their contribution to this progressive architecture direction.

REFERENCES

1. Mihailescu M., Horvath I.(1977). Velaroidal shells for covering universal industrial halls, Acta Techn. Acad. Sci. Hung.,**85**(1-2), pp. 135-145.
2. Hadid H.A. (1982). Analysis of parabolic velaroidal shells with simply supported boundary conditions,J. Struct. Eng., **8**(4), pp. 111-118.
3. Friaa Ahmed, Zenzri Hatem(1996). On funicular shapes in structural analysis and applications,Eur. J. Mech. A., **15**(5), pp. 901-914.
4. Shtayerman Yu. Ya., Bastatsky B.N.(1960).The Bending of Shallow Shells, M. – L.: Gosenergoizdat, 37 p.
5. Gogoberidze Ya. A. (1950). The Covering "Darbazi", Tbilisi, «Technica da shroma»,278 p.
6. Shambina S.L., Neporada V.I. (2012).Velaroidal surfaces and their application in building and architecture, Prazi TDATU, Iss. 4, vol. 53, pp. 168-173.
7. Bradshaw R., Campbell D., Gargari M., Mirmiran A., and Tripeny P. (2002). Special structures.Past, present, and future, Journal of Structural Engineering, June, 691-701.
8. Krivoshapko, S.N., Mamieva, I.A. (2012). Outstanding space structures of the last 20 years, Montazhn. ispetsial. raboti v stroitelstve, № 12, 8-14.

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