Study of a Queue Model Using an Estimation of Distribution Algorithm

Ricardo Pérez, S. Jöns, Arturo Hernández

Abstract*— *An analysis of a queue model M/M/1 for an outpatient clinic was done considering no-shows from the patients. In order to detect how no-shows affect the performance measure, i.e., the doctor's idle time on the patient-doctor system we consider analyzing the behavior of the patients when they have an appointment with a previous diagnostic successfully. The alternative approach was validated using a simulation model that was built through Delmia Quest® R20 and an Estimation of Distribution Algorithm to model the workflow in a small health clinic in México.

Index Terms*—*Queue Models, Appointment Scheduling, Estimation of Distribution Algorithms.

I. INTRODUCTION

Queue models have been studied and analyzed for many years. These have been used as a useful tool to model waiting line systems. M/M/1 a common queue model can be applied to different environments or scenarios such as patient-doctor. Traditionally, the M/M/1 represents the queue length in a system having a single server, where arrivals are determined by a Poisson process and service times have an exponential distribution.The patients are attended by a single doctor in order of arrival, FIFO as an example. When the patients can do an appointment, the arrival time can be determined in advance. A walk-in patients system is complex; it contains complex workflow and various uncertainties. Many constraints should be considered during the appointment scheduling phase. An M/M/1 queue model for walk-in patients does not consider no-shows (it is one of the assumptions in this model). In addition, the patients may arrive later than the scheduled appointment time or some patients could cancel the appointment. In this research, a study is carried out to improve the appointment scheduling on a small outpatient clinic in México. We analyzed the effect of no-show on the patient-doctor system through an Estimation of Distribution Algorithm (EDA) introduced by Mühlenbein and Paaβ [1] as an area of development in the field of evolutionary computation. We consider the situation on some patients do not come back to their appointments when the previous diagnostic was successfully. This situation has not been investigated in depth.

Discrete-Event Simulation models are widely applied to simulate the appointment scheduling problems in outpatient systems. In this paper, a discrete-event simulation model is built to simulate the workflow of the small health clinic; meanwhile the estimation of distribution algorithm calculates the conditional dependence among arrivals of the same patient to improve the performance measure, i.e., the doctor's idle time.

Manuscript Received November, 2013.
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II. PROBLEM STATEMENT

Multiple reasons may originate when some patient does not come back to his/her appointment. Normally, many of those reasons are no known by the medic. However, when the previous diagnostic was successfully, the next appointment could fail. In order to detect how this affect the performance measure, i.e., the doctor's idle time on the patient-doctor system we consider the conditional dependence among arrivals when the patients have an appointment with previous diagnostic successfully.

Table I depicts how the estimated probability (to attend an appointment by the patients) changes when the previous diagnostic was successfully on the small outpatient clinic mentioned. Tackling this situation we consider this probability to improve the appointment scheduling process on the clinic mentioned.

Table I. no–show probabilities on appointments.

	Appointment				
	I	II	III	IV	V
no-show probability	0.10	0.30	0.55	0.68	0.80

III. SIMULATION MODEL

In order to understand how no-shows affect the performance measure mentioned, a queue model was built in Delmia Quest® R20 simulation language. Our simulation model represents the real workflow on the clinic. We emulate different days with different appointments including no-shows as really it is. Parameters in the simulation model are listed as follows:

- Session duration. The opening hours of the clinic mentioned.
- Distribution of arrival. The interval between two consecutive arrivals determined by a Poisson process.
- No-show probability. The probability of the patients who do not show up on the day of their appointments when a previous diagnostic was successfully.
- Lateness rate. Ratio of the patients who arrive at the clinic later than their appointment time.
- Distribution of consult time. Time spent by the doctor in the consult room in each patient, considering exponential distribution.

The verification and validation model is the lengthiest step of the actual simulation model. First, animation screen together with dynamics statistics and graphs provided a general view of the system behavior. The researcher closely examined to verify whether the animation imitates the actual system. Second, in the case of face validity, a team consisting of 10 members evaluated the simulation model and their valuable comments helped to improve the model. Third, we reviewed the model by adjusting the amount of no-shows per week. The results of the simulation model utilized considering no-shows are shown in Table II.

Table II. Doctor's idle time considering several no-shows.

Patients	Simulation Model (min)
5	321.6
10	601.5
20	1045.5
50	2622.7
100	5568.6

IV. ESTIMATION OF DISTRIBUTION ALGORITHM

This kind of algorithm has been used satisfactorily to solve complex combinatorial optimization problems such as scheduling. Chen et al [2], Liu et al [3] and Pan and Ruiz [4] can be consulted.

The major procedure of an EDA is listed as follows.

Step 1. Set the generation index $g = 0$. Initialize an initial population $S^{(0)}$ of size *M*.

Step 2. Select a subset *D* from $S^{(g)}$ of size *N*, where $N \leq M$.

Step 3. Establish a probabilistic model *P* which somehow describes the distribution characteristics of *D*.

Step 4. Generate a set *K* of new individuals by sampling *P*.

Step 5. Select the best individuals from $K \cup S^{(g)}$ and assign them to the next generation $S^{(g+1)}$.

Step 6. Let g = g+1. If g<GN, where GN is the maximum number of generations return step 2. Otherwise, output the best solution in $S^{(g)}$.

Disadvantages of EDAs such as loss of diversity and insufficient use of location information of solutions have been tackled successfully by incorporating other methods such as Genetic Algorithms (GAs) during the evolutionary process. Chen et al [5] use this approach.

Various researches have been done in order to capture the problem structure with more precision. Advanced probabilistic models to solve scheduling problems through EDAs have been proposed attempting to integrate higher order interactions to enhance the solution quality. Wang et al [6] and Chen et al [7] have contributed on it.

Our approach is to use the MIMIC algorithm to build the probabilistic graph model. Introduced by De Bonet et al [8], the MIMIC algorithm uses a chain structured probabilistic model where the probability distribution of all the variables except the head node is conditioned on the value of the variable preceding them in the chain. It means a marginal univariate function and n-1 pairs of conditional density functions to build the probabilistic graph model.

A. Solution representation

Any solution of the problem mentioned should be a combination of a specific appointment for each patient and the estimation of no-show. Thus, a solution can be expressed by the start time of the appointment and the probability of no-show. In this paper, a solution is represented by two vectors (appointment scheduling and no-show estimation).

For the appointment scheduling vector, the number of elements is equal to the total number of patients to schedule, where each element contains a specific start time for the appointment. For the no-show estimation vector, each element represents the corresponding probability of no-show for each patient. To explain the representation, we provide an example by considering a problem with 5 patients as detailed in Table III.

Table III. Solution representation.

	Appointment scheduling vector					No-show estimation vector				
Hour	9	11	12	14	16	.21	.37	.76	.43	.29
Patient	1	2	3	4	5	1	2	3	4	5

B. Probability model

In this paper, the probability model is designed as two probability matrixes, i.e., appointment probability matrix and no-shows probability matrix.

The element $p_j(l)$ of the appointment probability matrix A_1 represents of the probability that patient *j* be scheduled in a specific hour of the appointment scheduling vector at generation *l*. For all *j* ($j = 1, 2, \ldots n$), p_j is initialized as $p_j(0) = 1/n$, where *n* represents the opening hours of the clinic, which ensures that the whole solution space can be sampled uniformly.

The element $q_{ji}(l)$ of the no-show probability matrix A_2 represents of the probability that patient *j* does not show up to his/her *i-th* appointment given that theprevious appointment has been successfully.

Via sampling according to the probability matrixes A_1 and A_2 new promising individuals may be generated.

C. Communication

We programmed the MIMIC algorithm on DevC++® and we integrate it to the simulation model in order to estimate the probabilities of no-show before to make the appointment. Figure 1 details the integration through a flow chart.

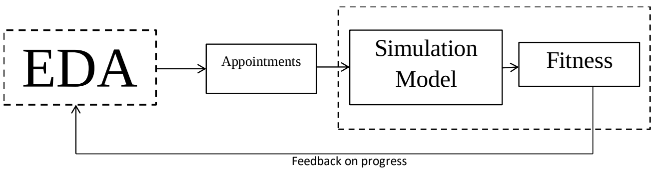

Fig. 1 EDA – simulation model integration.

V. RESULTS ANALYSIS

Table IV depicts how the EDA adjusts the appointment schedule considering some patients as no-shows. It helps to improve the performance measure after 50 trails.

Table IV. Appointment schedule adjusted.

Patients	Simulation Model (min)	EDA-Simulation Model (min)
5	321.6	284.1
10	601.5	590.3
20	1045.5	1010.5
50	2622.7	2598.6
100	5568.6	5437.4

Finally, an alternative appointment schedule scheme was built through the EDA in order to improve the performance measure.

VI. CONCLUSION AND FUTURE WORK

The approach proposed was used on a queue model for outpatient clinic. The results demonstrate there is a direct relation or interaction between previous diagnostic and the appointment. The EDA was able to identify this relationship in order to adjust the appointment schedule to improve the performance measure analyzed, up to 12%.

The future work is related on how we can get other relations or interactions among variables on queue models more sophisticated. Also different EDAs will be used in order tointegrate higher order interactions to enhance the solution quality.

ACKNOWLEDGMENT

We would like to express our gratefulness to Jon Fournier and Martin Barnes, for technical support on Quest® platform.

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