Extracting Inertial Parameters from Robotic Manipulators Dynamics

Galia V. Tzvetkova

Abstract—The paper presents theoretical bases and simulation results of an identification procedure to estimate inertial parameters of robotic manipulator dynamics. A proximity function is introduced and used as an adjustment element for adaptation of the procedure. The convergence of the estimation process is tested experimentally. The numerical results are shown after computer simulations over numerous robot trajectories.

Index Terms—Robot Parameters Identification, Recurssive estimation procedure.

I. INTRODUCTION

Parameter identification of dynamics supposes finding of numerical estimation of unknown parameters using input and output signal data of the object under investigation. Availability of *a priori* information for the parameters is important for the quality of the identification process. That means knowing the structure and the class of the model, which is going to describe the object. The conditions of real functioning of the object are also of significant importance. That includes the degree of uncertainty of wanted parameters, as well as the probability that external influence can disturb the process of identification. That demands adaptation of the identification process with the purpose of ensuring acceptable convergence rate and estimation accuracy of wanted parameters.

Contemporary computing resources permit multiple repetitions of codes for data processing. Recursive algorithms for parameter estimation use constant updating of incoming measurements of the investigated object and, as a result, gradually improve parameter evaluation.

In robotics, the following parameter identification algorithms have been concidered: off-line and on-line least-squares techniques [1], recursive least-squares and Kalman filtering [2], maximum likelihood identification method [3], genetic algorithms [4], neural networks [5], and others [6].

In this article, a recursive procedure for simultaneous estimation of inertial parameters of dynamic models of robot manipulators is proposed. The developed algorithm is adaptive with respect to the number of the iterations. Its convergence is verified using computer simulations over numerous robot trajectories.

The paper has the following structure: section 2 outlines the task to be solved, section 3 presents the philosophy of identification procedure, section 4 describes the plant model beeing identified, section 5 presents the recursive parameter estimation procedure itself, section 6 contains the simulation results for two degrees of freedom manipulation robot.

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Galia V. Tzvetkova works in the field of robotics.

II. OUTLINE OF THE TASK

The identification object is examined as a system described by its input-output signals. The schematic representation of the plant is shown in Figure 1.

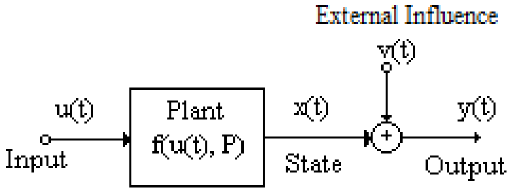

Figure1. Plant model

The plant contains unknown parameters, **P**. The plant parameters have to be estimated using output measurements $y(t)$, taking into consideration possible measurement disturbance or external influences $v(t)$.

The plant is examined as a system described by its input-output variables, represented generally as:

$$x(t) = f(u(t), \mathbf{p}) \tag{1}$$

where $\mathbf{P} = [p_1, p_2, ..., p_m]^{\mathrm{T}}$ is unknown parameter vector, $u(t)$ is control input, f is a known function, linear or non linear. If the unknown parameter **P** has constant in time entries, then the model (1) can be regarded as a stationary system of the kind [7]:

$$y(t) = x(t) + v(t) \tag{2}$$

where $x(t)$ is state variable, $v(t)$ is external influence variable, $y(t)$ is output variable of the plant.

The aim of the identification is to estimate the vector **P** for l consequent iterations using input $u(t)$ and output $y(t)$ variables of the plant (2). We choose a criterion for functioning of the identification process in the form of a function of the square of the difference between the real output $y(t)$ and the estimation of its state, $\hat{x}(t)$:

$$J(\hat{\mathbf{P}}^{(l)}) \equiv \frac{1}{2}[\mathbf{y}(t)^{(l)} - f(\mathbf{u}(t)^{(l)}, \hat{\mathbf{P}}^{(l)})]^2 \tag{3}$$

where $\hat{\mathbf{P}}^{(l)}$ is the estimation for the unknown parameters for the l-th iteration of the identification process.

We define a gradient of the criterion (3) with respect to the entries $\hat{p}_m$ of vector $\hat{\mathbf{P}}$:

$$\boldsymbol{\psi}^{(l)} \equiv \frac{\partial J(\hat{\mathbf{P}}^{(l)})}{\partial \hat{\mathrm{p}}_m} = \left[\frac{\partial J^{(l)}}{\partial \hat{p}_1}, \frac{\partial J^{(l)}}{\partial \hat{p}_2}, \ldots\ldots, \frac{\partial J^{(l)}}{\partial \hat{p}_m} \right]^{\mathrm{T}} \quad (4)$$

The recursive algorithm for evaluation of vector $\hat{\mathbf{P}}$ is received in the form [7]:

$$\hat{\mathbf{P}}^{(l+1)} = \hat{\mathbf{P}}^{(l)} - \rho^{(l)} \boldsymbol{\Psi}^{(l)}, \qquad l = 1,2,3,\ldots \quad (5)$$

where the function $\boldsymbol{\psi}^{(l)}$ is evaluated according to real output measurements $y(t)^{(l)}$ and the evaluated state $\hat{x}^{(l)}$ of the real process, $\rho^{(l)}$ is a sequence of scalar correcting coefficients.

III. CONSTRUCTION OF IDENTIFICATION PROCEDURE

The task for parameter identification of a dynamic plant is illustrated by generalized scheme in figure 2. The real dynamic process $y(t)$ depends on a vector $\mathbf{P}$ with unknown entries. The model $\hat{y}(t)$ of the real process depends on parameter vector $\hat{\mathbf{P}}$. The extent of identity of the real process and its model is evaluated by means of a computable criterion J, which generates, in turn, a sequence of parameters for adjustment of the entries of the vector $\hat{\mathbf{P}}$.

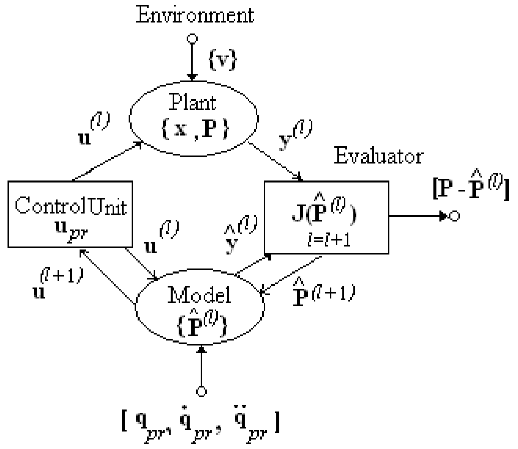

Figure 2. The estimation process

Let the real dynamic process be given by equation:

$$\dot{\mathbf{y}} = \Phi[\mathbf{y}, \mathbf{u}, \mathbf{P}, t] \quad (6)$$

where $\mathbf{u}$ is an input control vector.

The model of the process (6) is described similarly:

$$\dot{\hat{\mathbf{y}}} = \Phi[\hat{\mathbf{P}}, \mathbf{u}, \hat{\mathbf{y}}, t] \quad (7)$$

The function Φ in the model (7) is chosen in advance, and it is the same as for the real process. Vector $\hat{\mathbf{P}}$ can have a different dimension from the dimension of vector $\mathbf{P}$, and the number of entries of vector $\hat{\mathbf{P}}$ can be unknown in advance. It is assumed, however, that the vector $\hat{\mathbf{P}}$ belongs to a bounded space $\boldsymbol{\Omega}$,

$$\hat{\mathbf{P}} \in \boldsymbol{\Omega}, \quad (8)$$

and the bounds of $\boldsymbol{\Omega}$ are *a priori* given, thus defining a space, in which $\hat{\mathbf{P}}$ is wanted.

The identification procedure starts with choosing of a desired program motion $[\mathbf{q}_{pr}, \dot{\mathbf{q}}_{pr}, \ddot{\mathbf{q}}_{pr}]$ and the corresponding program control vector $\mathbf{u}_{pr}$, generated by the model. The initial value of the wanted vector parameter $\hat{\mathbf{P}}^{(0)}$ is chosen based on physical considerations about the plant, given in (8). The output of the procedure is the error of the estimation $[\mathbf{P} - \hat{\mathbf{P}}^{(l)}]$ which decreases with increasing of l. The process stops when the error becomes zero.

IV. ROBOT DYNAMICS PARAMETERIZATION

A. Inertial Parameters

Consider a robot manipulator of n serial linked rigid bodies. Coordinate systems $O_i(x_i, y_i, z_i)$ are attached to each link, $i = 1,2,\ldots,n$ in which the inertial parameters are defined. The inertial parameters with respect to centers of gravity of link i are:

m_i - mass of link i,

$(\mathrm{m}_i\bar{x}_i, \mathrm{m}_i\bar{y}_i, \mathrm{m}_i\bar{z}_i)$ - first moment of link i,

$(\bar{x}_i, \bar{y}_i, \bar{z}_i)$ - coordinates of the center of gravity in coordinate system O_i,

$(\mathrm{I}_{xxi}, \mathrm{I}_{xyi}, \mathrm{I}_{xzi}, \mathrm{I}_{xyi}, \mathrm{I}_{yzi}, \mathrm{I}_{zzi})$ inertial parameters, computed with respect to vertical axis z_i of the coordinate system i.

For each link, 10 elements vector of inertial parameters $\mathbf{P}^{\mathrm{i}}$ is formed:

$$\mathbf{P}^{\mathrm{i}} = [\mathrm{I}_{xxi}\mathrm{I}_{xyi}\mathrm{I}_{xzi}\mathrm{I}_{yyi}\mathrm{I}_{yzi}\mathrm{I}_{zzi}\,(\mathrm{m}_i\bar{\mathrm{x}}_i)(\mathrm{m}_i\bar{\mathrm{y}}_i)(\mathrm{m}_i\bar{\mathrm{z}}_i)\,\mathrm{m}_i]^{\mathrm{T}} \quad 9)$$

The maximum number of inertial parameters for a robot with n degrees of freedom is a 10n vector [8]:

$$\begin{aligned} \mathbf{P} &= [\mathbf{P}^1\ \mathbf{P}^2 \ldots \mathbf{P}^{\mathrm{i}} \ldots \mathbf{P}^{\mathrm{n}}]^{\mathrm{T}} \\ &= [\mathrm{p}_1^1 \ldots \mathrm{p}_{10}^1\ \mathrm{p}_{11}^2 \ldots \mathrm{p}_{20}^2 \ldots \mathrm{p}_{\mathrm{j}}^{\mathrm{i}} \ldots \mathrm{p}_{10\mathrm{n}}^{\mathrm{n}}]^{\mathrm{T}} \quad (10) \\ &\mathrm{i} = 1,\mathrm{n} \ \ ,\ \ \mathrm{j} = 1,10\mathrm{n} \end{aligned}$$

B. The Identification Model

Let examine the dynamic process, described by the model:

$$\mathbf{D}(\mathbf{q}, \mathbf{P})\ddot{\mathbf{q}} + \mathbf{C}(\mathbf{q}, \dot{\mathbf{q}}, \mathbf{P}) = \boldsymbol{\tau} + \mathbf{v} \quad (11)$$

where $\mathbf{q}, \dot{\mathbf{q}}, \ddot{\mathbf{q}}$ are $(n \times 1)$ vector-functions of general coordinates, velocities and accelerations, $\mathbf{D}(\mathbf{q}, \mathbf{P})$ $(n \times n)$ is a symmetric, positive definite matrix, $\mathbf{C}(\mathbf{q}, \dot{\mathbf{q}}, \mathbf{P})$ $(n \times 1)$ vector-function of general coordinates and velocities, $\boldsymbol{\tau}$ $(n \times 1)$ vector-function of generalized forces, $\mathbf{v}$ - is an external influence.

Based on the fact that the matrix functions $\mathbf{D}(\mathbf{q}, \mathbf{P})$ and $\mathbf{C}(\mathbf{q}, \dot{\mathbf{q}}, \mathbf{P})$ depends linearly on the vector of unknown parameters $\mathbf{P}$, the equation of the dynamic process (11) can be transformed as [9]:

$$\mathbf{G}(\mathbf{q},\dot{\mathbf{q}},\ddot{\mathbf{q}})\mathbf{P}=\boldsymbol{\tau} \qquad (12)$$

or

$$\mathbf{GP}=\mathbf{u} \qquad (13)$$

where $\mathbf{P}$ is $(m\times 1)$ vector of unknown parameters, $\mathbf{u}=f(\boldsymbol{\tau})$ is $(n\times 1)$ input control vector, $\mathbf{G}$ is $(n\times m)$ matrix of measurements, known as regression or design matrix.

For the purpose of parametric identification of the robot dynamics, the entries of vector $\mathbf{P}$ should be numerically estimated.

V. RECURSIVE ESTIMATION OF LINEAR MODEL

A. The least square solution

Let's take the control vector in (13) in the form:

$$\mathbf{u}=\mathbf{G}(\mathbf{q},\dot{\mathbf{q}},\ddot{\mathbf{q}})\mathbf{P} \qquad (14)$$

We can express any real trajectory as:

$$\mathbf{q}_*(t)=\ddot{\mathbf{q}}_{pr}(t)-\alpha[\dot{\mathbf{q}}(t)-\dot{\mathbf{q}}_{pr}(t)]-\beta[\mathbf{q}(t)-\mathbf{q}_{pr}(t)] \qquad (15)$$

$\alpha,\beta\succ 0$

We calculate the control vector for the above trajectory as:

$$\mathbf{u}=\mathbf{G}(\mathbf{q},\dot{\mathbf{q}},\mathbf{q}_*)\hat{\mathbf{P}} \qquad (16)$$

where $\mathbf{q}_{pr}(t)$, $\dot{\mathbf{q}}_{pr}(t)$, $\ddot{\mathbf{q}}_{pr}(t)$ is the program trajectory, velocity and acceleration, $\hat{\mathbf{P}}$ is an estimation for the vector of wanted parameters.

The control equations (14) and (16) give:

$$\mathbf{GP}=\mathbf{G}_*\hat{\mathbf{P}} \qquad (17)$$

where $\mathbf{G}_*=\mathbf{G}(\mathbf{q},\dot{\mathbf{q}},\mathbf{q}_*)$.

If the columns of $\mathbf{G}$ are linearly independent, then the equation (16) has unique solution for **P**:

$$\mathbf{P}=\mathbf{Q}\hat{\mathbf{P}} \qquad (18)$$

$$\mathbf{Q}=\mathbf{G}^+\mathbf{G}_*$$

$$\mathbf{G}^+=(\mathbf{G}^{\mathrm{T}}\mathbf{G})^{-1}\mathbf{G}^{\mathrm{T}} \qquad (19)$$

where $\mathbf{G}^+$ is pseudo – inverse according to Moore – Penrose. If $\mathbf{G}^+$ does not exist, i.e. the columns of $\mathbf{G}$ are linearly dependent**,** than the recursive algorithm of next section is applied.

B. The Recursive Algorithm

Consider the difference between the real and the program motions:

$$\boldsymbol{\eta}=\ddot{\mathbf{q}}-\ddot{\mathbf{q}}_{pr}^{(l)}+\alpha(\dot{\mathbf{q}}-\dot{\mathbf{q}}_{pr}^{(l)})+\beta(\mathbf{q}-\mathbf{q}_{pr}^{(l)}) \qquad (20)$$

Then the control input $\mathbf{u}^{(l)}$ is constructed based on (15) and by using the plant model (11):

$$\mathbf{u}^{(l)}=\mathbf{C}(\mathbf{q},\dot{\mathbf{q}},\hat{\mathbf{P}}^{(l)})-\mathbf{D}(\mathbf{q},\hat{\mathbf{P}}^{(l)})[-\ddot{\mathbf{q}}_{pr}^{(l)}+\alpha(\dot{\mathbf{q}}-\dot{\mathbf{q}}_{pr}^{(l)}+\beta(\mathbf{q}-\mathbf{q}_{pr}^{(l)})] \qquad (21)$$

where $\hat{\mathbf{P}}^{(l)}$ is an estimation of the vector $\mathbf{P}$, l is the successive iteration of identification procedure.

The gradient of input control vector (21) with respect to the vector of the unknown parameters $\hat{\mathbf{P}}^{(l)}$ is:

$$\boldsymbol{\Psi}^{(l)}=\nabla_{\hat{\mathbf{P}}}\{\mathbf{C}(\mathbf{q},\dot{\mathbf{q}},\hat{\mathbf{P}}^{(l)})-\mathbf{D}(\mathbf{q},\hat{\mathbf{P}}^{(l)})[\boldsymbol{\eta}-\ddot{\mathbf{q}}]\} \qquad (22)$$

We introduce the proximity function:

$$\mathbf{PRX}^{(l+1)}=(\boldsymbol{\psi}^{(l+1)})^{\mathrm{T}}\boldsymbol{\psi}^{(l+1)}\hat{\mathbf{P}}^{(l)}+(\boldsymbol{\psi}^{(l+1)})^{\mathrm{T}}\mathbf{u}^{(l)} \qquad (23)$$

The recursive algorithm for estimation of vector $\hat{\mathbf{P}}^{(l)}$ according to (5) is given by:

$$\hat{\mathbf{P}}^{(l+1)}=\hat{\mathbf{P}}^{(l)}-\frac{\lambda^{(l+1)}\sigma_D}{\left\|\boldsymbol{\Psi}^{(l+1)}\right\|^2}(\boldsymbol{\Psi}^{(l+1)})^{\mathrm{T}}\boldsymbol{\eta}^{(l+1)} \qquad (24)$$

where σ_D is the minimal eigenvalue of the matrix $\mathbf{D}(\mathbf{q},\hat{\mathbf{P}}^{(l)})$,

$$\lambda^{(l+1)}=\begin{cases}1, & \text{if } \mathbf{PRX}^{(l+1)}\neq 0\\ 0, & \text{if } \mathbf{PRX}^{(l+1)}=0\end{cases}$$

$\left\|\boldsymbol{\Psi}\right\|^2=Sp(\boldsymbol{\Psi}\boldsymbol{\Psi}^{\mathrm{T}})$, $\varepsilon_1,\varepsilon_2\succ 0$ are given small numbers.

C. Adaptation and Convergence of the Iterations

The function $\mathbf{PRX}^{(l+1)}$ is the adaptive element of the recursive estimation process (24). The iterations stop when the following equality is fulfilled:

$$\mathbf{PRX}^{(l+1)}=0$$
$$l=\mathrm{NA}+1,\ \mathrm{NA}+2,\ \mathrm{NA}+3,\ldots\ldots \qquad (25)$$

where NA is the number of the iteration when the equality (25) is fulfilled for the first time.

This means that $\hat{\mathbf{P}}^{(l+1)}=\hat{\mathbf{P}}^{(l)}=\mathbf{P}$, i.e. the estimation of the wanted parameters remains constant and coincide with the actual values of the plant parameters.

VI. SIMULATION RESULTS

Two degrees of freedom manipulator working in the horizontal plane is described by matrix equation:

$$\begin{bmatrix}u_1\\u_2\end{bmatrix}=\begin{bmatrix}D_{11} & D_{12}\\ D_{21} & D_{22}\end{bmatrix}\begin{bmatrix}\ddot{q}_1\\ \ddot{q}_2\end{bmatrix}+\begin{bmatrix}0 & C_{122}\\ C_{211} & 0\end{bmatrix}\begin{bmatrix}\dot{q}_1^2\\ \dot{q}_2^2\end{bmatrix}+\begin{bmatrix}C_{112}\\ 0\end{bmatrix}[\dot{q}_1\dot{q}_2] \qquad (26)$$

where $[u_1,u_2]^{\mathrm{T}}$ is the control input vector,

$D_{11}=\mathrm{I_{xx1}}+\mathrm{I_{yy1}}+\mathrm{L_1^2m_1}+2\mathrm{L_1(m_1\bar{x}_1)}+\mathrm{I_{xx2}}+\mathrm{I_{yy2}}$
$+(2\mathrm{c_2L_1L_2}+\mathrm{L_1^2}+\mathrm{L_2^2})\mathrm{m_2}$
$+(2\mathrm{c_2L_1}+2\mathrm{L_2})(\mathrm{m_2\bar{x}_2})-2\mathrm{L_1s_2(m_2\bar{y}_2)}$

$D_{12}=\mathrm{I_{xx2}}+\mathrm{I_{yy2}}+(\mathrm{c_2L_1L_2}+\mathrm{L_2^2})\mathrm{m_2}$
$+(\mathrm{c_2L_1}+2\mathrm{L_2})(\mathrm{m_2\bar{x}_2})-\mathrm{L_1s_2(m_2\bar{y}_2)}$

$D_{21}=D_{12}$

$D_{22}=\mathrm{I_{xx2}}+\mathrm{I_{yy2}}+\mathrm{L_2^2m_2}+2\mathrm{L_2(m_2\bar{x}_2)}$

$C_{122}=C_{211}=\mathrm{s_2L_1L_2m_2}+\mathrm{s_2L_1(m_2\bar{x}_2)}+\mathrm{c_2L_1(m_2\bar{y}_2)}$

$\mathrm{L_1,L_2}$ lengths of the links.

c_2, s_2 stand for $\cos(q_2)$ and $\sin(q_2)$.

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The control vector $\mathbf{u}$ is reconstructed according to (14):

$$\mathbf{u} = \mathbf{G}(\mathbf{q}, \dot{\mathbf{q}}, \ddot{\mathbf{q}})\mathbf{P}$$

where the regression matrix **G** is takes the form:

$$\mathbf{G} = \begin{bmatrix} \ddot{q}_1 & (\ddot{q}_1 + \ddot{q}_2) & (2c_2\ddot{q}_1 + c_2\ddot{q}_2 - 2s_2\dot{q}_1\dot{q}_2) & s_2\dot{q}_1^2 & (-2s_2\ddot{q}_1 - s_2\ddot{q}_2 - c_2\dot{q}_2^2 - 2c_2\dot{q}_1\dot{q}_2) \\ 0 & (\ddot{q}_1 + \ddot{q}_2) & c_2\ddot{q}_1 & -s_2\dot{q}_2^2 & -s_2\ddot{q}_1 + c_2\dot{q}_1^2 \end{bmatrix}$$

and the parameter vector $\mathbf{P} = [p_1, p_2, ..., p_5]^T$ has the entries [10]:

$$p1 = I_{zz1} + 2L_1(m_1\bar{x}_1) + L_1^2 m_1 + L_1^2 m_2$$
$$p2 = I_{zz2} + 2L_2(m_2\bar{x}_2) + L_2^2 m_2$$
$$p3 = L_1(m_2\bar{x}_2) + L_1 L_2 m_2$$
$$p4 = L_1(m_2\bar{y}_2) + L_1 L_2 m_2$$
$$p5 = L_1(m_2\bar{y}_2) \qquad (27)$$

The desired program motion is chosen as:

$$\mathbf{q}_{np} = \left[q_{1pr}(t)\ \ q_{2pr}(t)\right]^T$$
$$q_{1pr}(t) = A_1(a_1t^5 + a_2t^4 + a_3t^3) \qquad (28)$$
$$q_{2pr}(t) = A_2(a_1t^5 + a_2t^4 + a_3t^3)$$

where A_1, A_2, a_1, a_2, a_3 are appropriately chosen coefficients. The program motions for the two links are depicted in Figure 3(a), 3(b).

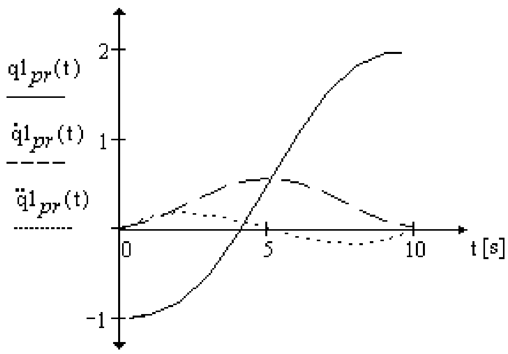

Figure 3(a). The program movement for link 1

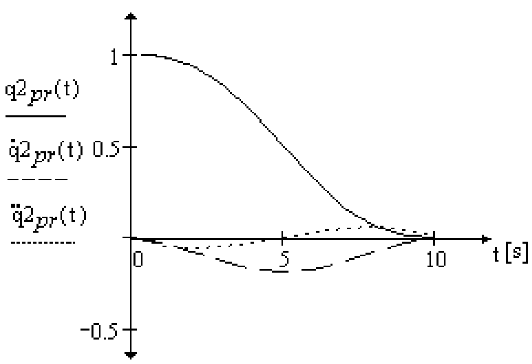

Figure 3(b). The program movement for link 2

The program control corresponding to the program motion (28) together with the successive iterations of the control vectors (21) are shown in Figure 4(a), 4(b).

The successive iterations of the entries of parameter vector $\hat{\mathbf{P}}^{(l)}$ (24), shown in Figure 5, tend to steady state values $\mathbf{P} = [22, 5, 3, 2, 0.1]^T$ for 100 iterations procedure.

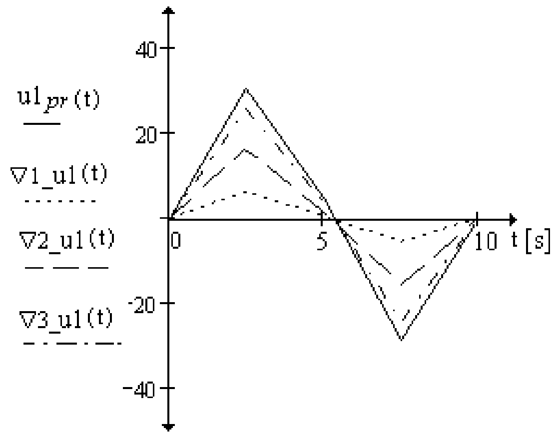

Figure 4(a). The gradient of u1

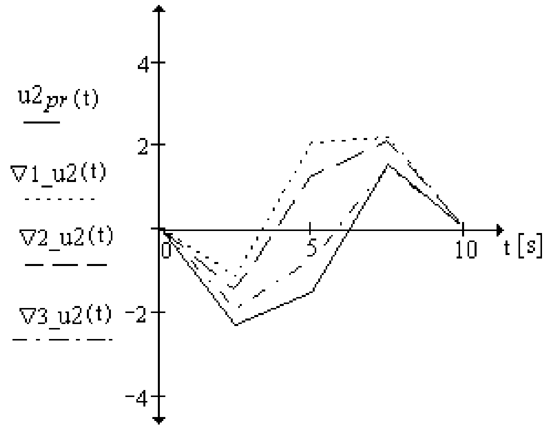

Figure 4(b). The gradient of u2

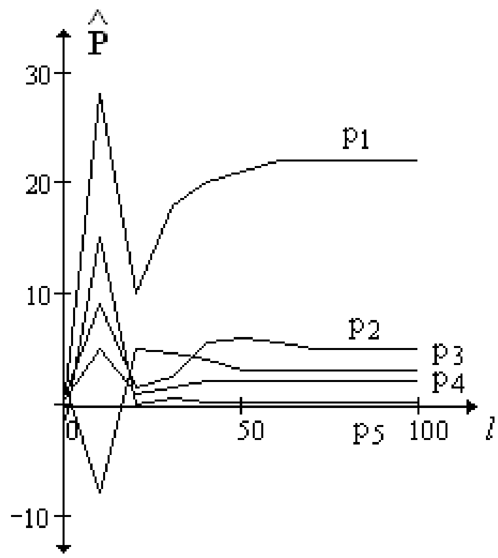

Figure 5. The estimated vector $\hat{\mathbf{P}}$

VII. CONCLUSION AND FUTURE WORK

This paper presents a recursive identification procedure for simultaneous estimation of robot manipulators dynamics parameters. The algorithm is developed based on proposed recursive estimatin process. A function of proximity is introduces to verify the convergence of the estimation procedure. Further efforts will be directed towards identification of robots with higher degrees of freedom.

REFERENCES

1. D. Smallwood, L. Whitcomb, Adaptive Identification of Dynamically Positioned Underwater Robotic Vehicles, *IEEE Transactions on Control Systems Technology*, Vol.11, No. 4, 2003, pp. 505-515
2. V. Sujan, S. Dubowsky, An Optimal Information Method for Mobile Manipulator Dynamic Parameter Identification, *IEEE/ASME Transactions on Mechatronics*, Vol.2, No.2, 2003, pp. 215-225.
3. **M. Olsen, J. Swevers, W. Verdonck,** Maximum Likelihood Identification of a Dynamic Robot Model: Implementation Issues, *The International Journal of Robotics Research*, Vol. 21, No.2, 2002, pp.89-96.
4. A. Nemes, B. Lantos, Genetic Algorithm-Based Optimization of Fuzzy Logic Systems for Dynamic Modeling of Robots, *Periodica Polytechnica Ser. El. Eng*. Vol. 43, No. 3, 1999, pp. 177–187.
5. Y. Maeda, R. J. P. De Figueiredo, Learning Rules for Neuro-Controller via Simultaneous Perturbation, *IEEE Transactions On Neural Networks*, Vol.8, No.5, 1997, pp.1119-1130.
6. K. Yoshida, N. Ikeda, H. Mayeda, Experimental Study of the Identification Methods for the Industrial Robot Manipulator, *JRSJ* Vol.11, No.4, pp.84-93.
7. J. C. Spall, Adaptive Stochastic Approximation by the Simultaneous Perturbation Method, *IEEE Transactions on Automatic Control*, Vol. 45, No. 10, 2000, pp. 1839-1853
8. W.Khalil, M.Gautier, Calculation of the Identifiable Parameters for Robots Calibration, *9th IFAC/IFORS Symposium on Identification and System Parameter Estimation*, Budapest, 1991, pp. 888-892.
9. A. Goldenberg, J. Apkarian and H. Smith, An Approach to Adaptive Control of Robot Manipulators Using the Computed Torque Technique, *ASME Journal of Dynamic Systems, Measurement, and Control*, Vol.111, 1989, pp. 1-8.
10. G. Tzvetkova, Symbolic Reconstruction of Robot Dynamic Model by Using Matrix Transformations, *Proceedings of the IASME International Conference on MECHANICS and MECHATRONICS*, UDINE, Italy, March 25-27, CD, 2004, pp.155-159.