

Asymptotic Power Limits for Ad-Hoc Networks Reconsider the Smart Antenna Cases

K. Venkatraman, Sudhir .P. J. Raj, R. AntoRose, K. Pasupathi, S. Manikandan

Abstract— Smart antennas can be useful in significantly increasing the capacity of wireless ad hoc networks. A number of media access and routing protocols have been recently proposed for the use with such antennas, and have shown significant performance improvements over the omnidirectional case. However, none of these works explores if and how different directional and smart antenna designs affect the asymptotic capacity bounds, derived by Kumar and Gupta[12]. These bounds are inherent to specific ad-hoc network characteristics, like the shared wireless media and multi-hop connectivity, and pose major scalability limitations for such networks. In this work, we present how smart antennas can affect the asymptotic behavior of an ad-hoc network's capacity. Specifically, we perform a capacity analysis for an ideal flat-topped antenna, a linear phased-array antenna, and an adaptive array antenna model. Finally, we explain how an ad-hoc network designer can manipulate different antenna parameters to improve the scalability of an ad-hoc network.

Keywords-capacity; ad-hoc; directional antennas; smart antennas;

I.INTRODUCTION

Wireless ad-hoc networks are multi-hop networks where all nodes cooperatively maintain network connectivity. The ability to be set up fast and operate without the need of any wired infrastructure (e.g. base stations, routers, etc.) makes them a promising candidate for military, disaster relief, and law enforcement applications. Furthermore, the growing interest in sensor network applications has created a need for protocols and algorithms for large-scale self-organizing ad-hoc networks, consisting of hundreds or thousands of nodes. Until recently, it was commonly assumed that nodes in ad-hoc networks are equipped with omnidirectional antennas.

However, during the past couple of years there has been a rapidly growing interest in the use of directional or smart antennas in ad-hoc networks [2] [3] [17]. Such antennas have the ability to concentrate the radiated power towards the intended direction of transmission or reception. As a result of this, they can help reduce the amount of radiated power necessary to reach a node, and that way greatly improve the energy efficiency of ad-hoc network protocols [16] [17]. Furthermore, smart/adaptive antennas can minimize interference by steering the antenna pattern nulls towards the sources of interference. Therefore, a higher number of simultaneous transmissions could be sustained by the network.

Manuscript received January 15, 2014.

K. Venkatraman, Veltech Multi Tech Dr.Rangarajan Dr.Sakunthala Engineering College, Avadi, Chennai, India.

Sudhir .P. J. Raj, Veltech Multi Tech Dr.Rangarajan Dr.Sakunthala Engineering College, Avadi, Chennai, India

R. Anto Rose, Veltech Multi Tech Dr.Rangarajan Dr.Sakunthala Engineering College, Avadi, Chennai, India.

K. Pasupathi, Veltech Multi Tech Dr.Rangarajan Dr.Sakunthala Engineering College, Avadi, Chennai, India.

S. Manikandan, Veltech Multi Tech Dr.Rangarajan Dr.Sakunthala Engineering College, Avadi, Chennai, India.

By designing appropriate communication protocols that exploit the potential of directional and smart antennas one could significantly improve the capacity, throughput and end-to-end delay of wireless ad-hoc networks, [2] [7]].

An important characteristic of a wireless ad-hoc network (or as a matter of fact any network in general) is its capacity. A number of recent papers has explored several issues related to the capacity of ad-hoc networks for both the cases where nodes are assumed to have omnidirectional antennas [11] [13] [14], as well as directional or smart antennas. However, specific assumptions are being made in each of the above papers in terms of the technologies and protocols used by nodes, in order to model current practice in ad-hoc networks. In this respect, the relevant capacity analysis and results are technology-dependent, that is, they hold for the specific scenarios being modeled. In a more recent work, Xie and Kumar take a more information-theoretic approach, in order to derive scaling laws for the capacity of wireless networks, that hold regardless of specific technologies and protocols used [23].

Probably the most well-known and defining result for the capacity of ad-hoc networks is given by Kumar and Gupta [11]. In this work, the authors prove that in a multi-hop wireless network, where nodes are randomly placed on a plane and each node chooses a destination node in random, the capacity available to each node, for its own traffic,

decreases as a function of $O(1/\sqrt{n \log n})$, n being the total number of nodes in the network. This result holds, regardless of whether multiple channels are used. It is a scaling law that stems from two intrinsic characteristics of ad-hoc networks, namely their multi-hop nature and the need for nodes to compete for the shared wireless media. The former implies that for each packet generated in the network, a growing number of intermediate nodes need to be involved in forwarding that packet from sender to destination, with increasing n , creating a higher per node overhead. The latter means that there is a restriction on the number of simultaneous transmissions at any time. The important and somewhat discouraging implication of this result, as the authors themselves note, is that there is an inherent scalability limitation for ad-hoc networks that should make designers target their efforts at designing only small networks.

In this paper we present how the use of directional and smart antennas can affect the asymptotic capacity behavior of ad-hoc networks and improve their scalability. In Section II, we discuss the directional and smart antenna models that we've used for our analysis. In Section III, we derive relations between the order of capacity growth and specific antenna parameters, like number of antenna elements, beam-width and main-lobe/side-lobe gain ratio. We use these relations in Section IV, in order to explore how simple antenna parameter manipulation can allow the ad-hoc network



designer to improve the scaling law order of ad-hoc networks.

II. ANTENNA MODELS

A. Smart Antenna Models

In this work, we are interested more in adaptive array antennas that can independently steer their main beam and nulls to arbitrary directions. This process is generally called beam forming. Their main difference from simple directional antennas (hence their smartness) is the following: Instead of just directing the main beam towards the direction specified (e.g. by the application), smart antennas can automatically adapt their radiation pattern¹, in order to track the intended receiver/transmitter and minimize transmission/reception gain (i.e. create nulls) towards unintended receivers/transmitters. A large number of alternative beam forming designs (e.g. digital, microwave, aerial beam forming) and algorithms (e.g. Least Mean Square, Constant Modulus Algorithm, etc.) have been proposed, a detailed tutorial of which can be found in [1].

Until recently, adaptive array antennas had only been considered for the use on base station in cellular systems, due to their large size, increased cost and power consumption, and complexity of design. However, there have recently been proposed simple, analog, smart antenna designs [4] [5] that are low cost and energy-efficient enough to be used on wireless terminals. They're based on the concept of aerial beam-forming and prototypes have been built and tested [6].

An adaptive array antenna consisting of N elements is said to have N-1 degrees of freedom. Without any detail on how this is done, this roughly implies that such an antenna can independently track one node of interest and cancel N-2 non-coherent interferers. In our subsequent analysis we assume that a smart/adaptive antenna of N elements can turn its main beam of gain $G_{max}=1$ to an arbitrary direction while creating nulls of gain $G_{null} \ll 1$ towards at most N-2 different directions².

B. Protocol and Physical Model

As mentioned earlier, the need for all nodes to share the common wireless media implies that there is a limit on the number of simultaneous transmissions that can successfully occur at any time. This limit could be dictated by some media access control (MAC) protocol that spaces concurrent transmissions far enough from each other, so as to guarantee avoidance of most or all collisions (e.g. CSMA/CA). Alternatively, this limit may be imposed by the physical properties of the media. Specifically, one could assume that any set of simultaneous transmissions is permissible, as long as the SINR (Signal to Noise and Interference Ratio) at each receiver is above a specific threshold β . The above two models, namely the Protocol Model and the Physical Model, respectively, were first introduced in [11] for analyzing the capacity of wireless networks, when omni-directional antennas are used. We adapt these models for the cases of directional and smart antennas. We assume a two-ray ground propagation model and let P be the common transmitting power of all nodes, P_{th} the receiving power threshold and h the antenna height. We present asymptotic capacity results for three representative scenarios/cases:

C. Smart Antenna & Protocol Model

All nodes are assumed to be equipped with an adaptive array antenna of N elements. A media access protocol resolves simultaneous transmission request, such that within a range $(1+\Delta)R_3$ from any receiving node, at most N-2 other nodes may be receiving at the same time. R_3 is given by

$$R_3 = \left(\left(P_t / P_{th} \right) h^2 \right)^{1/4} \quad (5)$$

III. ASYMPTOTIC CAPACITY ANALYSIS

In this section we present the asymptotic capacity laws, originally derived by Kumar and Gupta for the case of omnidirectional antennas [11], appropriately modified for the three scenarios outlined in section 2. All proofs are based on the capacity analysis followed in [11], modified to incorporate appropriate antenna parameters into the equations. Due to limitations in space, we only present the proof for case 1 in the Appendix. Additionally, we are mainly concerned with the asymptotic behavior of the capacity equations. Therefore, all linear scaling factors, besides antenna parameters of interest, are captured in appropriate constants c_1 , c_2 , and c_3 . We summarize here our assumptions:

- i) There are n nodes randomly distributed on a planar disk of unit area. If the size of the disk is A, instead, then all results need to be scaled by \sqrt{A} , as explained in [11].
- ii) Each node randomly picks a destination node for its traffic. The average distance \bar{L} between sender-destination nodes is $O(1)$.
- iii) The network transports λnT bits over a period of T seconds, where λ denotes the average transmission rate for each node to its destination over a period T.
- iv) For simplicity, we assume that there is only a single wireless channel of capacity W bits/sec, available to all nodes. All results hold also for the case of multiple channels, whose aggregate capacity is equal to W.

Smart Antennas & Protocol Model

When smart antennas are used on each node, the analysis is the same as the original one for the omni-directional antenna case [11], with only the following difference: Each receiving node creates a silence region of disk shape around it. However, up to N-2 additional nodes in that disk may be receiving simultaneously. Hence, the resulting bound for λ is scaled by a factor proportional to N-2 as follows:

$$\lambda(n) \leq \frac{c_3 W}{\sqrt{n \log n}} (N-2) \quad (8)$$

IV. IMPROVING THE SCALING LAWS

As we can see by equations (6), (7), and (8), we have expressed the asymptotic capacity bounds for all three cases, as functions of different antenna parameters, like number of elements, antenna gain and beam width. The importance of those results is easier seen from an ad-hoc network designer's perspective. Let us view all relevant antenna parameters as different functions of n, namely $N(n)$, $\bar{G}_I(n)$, $G_{side}(n)$ and $\theta(n)$, where n is the number of nodes. This does not necessarily mean that we assume antennas can dynamically modify their parameters. It merely implies



that the designer can make its choice of directional or smart antenna parameters to be used on nodes, based on the expected scale of the ad-hoc network. For example, if a designer chooses to scale the number of elements N in a smart antenna, as a function of $\Theta(\sqrt{\log n})$ it would improve the scaling order of $\lambda(n)$ (see Eq. 8) from $O(1/\sqrt{\log n})$ to $O(1/\sqrt{n \log n})$ or $O(1/\sqrt{n})$. This allows an asymptotically increased number of nodes in the network to sustain a specific per node transmission rate.

Of course, one should be aware of that antenna parameters like number of elements, gain, and beam-width cannot be increased at will. This could require technologies and designs that would be conflicting with the requirement for simple, inexpensive, low-energy antennas for wireless terminals. Therefore, it is quite interesting to see how feasible different scaling requirements are, for the different antenna models we've assumed. We will do so, through two examples.

Let us consider the previous example of scaling the number of elements N in a smart antenna, as a function of $\Theta(\sqrt{\log n})$. We already saw how this approach affects the scalability of the network. Now we examine what this requirement implies in practice, for N . Let's assume that the scale of the ad-hoc network changes from n_1 to n_2 nodes. The relative increase in the number of antenna elements is given by $N_r = \sqrt{\log n_2 / \log n_1}$ and its value is shown in Table.2 for different values of n_1 and n_2 . As we can see, the relative increase is small enough to be feasible for practical smart antennas. Finally, note that the relative increase in N per order of magnitude growth in network size becomes smaller for larger networks.

RELATIVE INCREASE IN NUMBER OF ELEMENTS

N_1	100	10	100
N_2	1000	100	10000
N_r	1.45	1.418	1.42

V.CONCLUSIONS

In this paper we have analyzed how the use of smart antennas affects the asymptotic capacity behavior of wireless ad-hoc networks. We performed our analysis for an ideal flat-topped antenna model, as well as two realistic antenna models, namely a linear phased-array antenna, and an adaptive array antenna. We used two different models for the access to the wireless media, namely the Physical and Protocol model, and combined them with the above three antenna models to derive asymptotic capacity equations that incorporate appropriate antenna parameters. Finally, we have shown how the use of directional and smart antennas can alleviate the intrinsic scalability limitations of wireless ad-hoc networks.

REFERENCES

1. W. F. Gabriel, "Adaptive Processing Array Systems," in Proceedings of IEEE, Vol. 80, Issue 1, Jan 1992.
2. Asymptotic power limits for Ad-hoc Networks reconsider the e Directional and Smart Antenna Cases
3. Bellofiore, J. Foutz, R. Govindarajula, I. Bahceci, C.A. Balanis, A.S. Spanias, J.M. Capone, and T.M. Duman, "Smart antenna system analysis, integration and performance for mobile ad-hoc networks

- (MANETs)," IEEE Transactions on Antennas and Propagation, Volume: 50 Issue: 5, May 2002 Page(s): 571 –581
4. R. Radhakrishnan, D. Lai, J. Caffery, and D.P Agrawal, "Performance comparison of smart antenna techniques for spatial multiplexing in wireless ad hoc networks," The 5th International Symposium on Wireless Personal Multimedia Communications, 2002, Volume: 1, 2002. Page(s):168-171.
5. T. Ohira, "Analog smart antennas: an overview," The 13th IEEE International Symposium on Personal, Indoor and Mobile Radio Communications, 2002, Volume: 4, 2002, Page(s): 1502 –1506.
6. T. Ohira, "Blind adaptive beamforming electronically-steerable parasitic array radiator antenna based on maximum moment criterion," in IEEE Antennas and Propagation Society International Symposium, 2002, Vol. 2, 2002, Page(s): 652 –655.
7. T. Ohira, and K. Gyoda, "Electronically steerable passive array radiator antennas for low-cost analog adaptive beamforming," in Proceedings of IEEE International Conference on Phased Array Systems and Technology 2000, Page(s): 101 –104.
8. Nasipuri, S. Ye, J. You, and R. E. Hiromoto, "A MAC protocol for mobile ad hoc networks using directional antennas," IEEE Wireless Communications and Networking Conference (WCNC'2000), 2000.
9. M. Takai, J. Martin, R. Bagrodia, and A. Ren, "Directional Virtual Carrier Sensing for Directional Antennas in Mobile Ad Hoc Networks," Proc. ACM MobiHoc '2002, June 2002
10. R. Roychoudhury, X. Yang, R. Ramanathan, and N. Vaidya, "Medium Access Control in Ad Hoc Networks Using Directional Antennas," in Proc. of MOBICOM '2002, September 2002.
11. L. Bao, and J.J. Garcia-Luna-Aceves, "Transmission Scheduling in Ad Hoc Networks with Directional Antennas," in Proc. of ACM/IEEE MOBICOM '2002, September 2002.
12. P. Gupta, and P. R. Kumar, "The capacity of wireless networks," IEEE Transactions on Information Theory, vol. 46, pp. 388-404, March 2000.
13. Spyropoulos, and C. S. Raghavendra, "Capacity Bounds for Ad-hoc Networks using Directional Antennas," to appear in Proc. IEEE ICC '2003, May 2003.
14. J. Li, C. Blake, D. S. J. Decouto, H. I. Lee, and R. Morris, "Capacity of Wireless Ad Hoc Networks," in Proc. MOBICOM '2001, July 2001.
15. S. Toumpis, and A.J. Goldsmith, "Capacity Regions for Wireless Ad Hoc Networks," in Proc. IEEE ICC '2002, May 2002.
16. Ram Ramanathan, "On the Performance of Beamforming Antennas in Ad Hoc Networks", Proc. of the ACM/SIGMOBILE MobiHoc 2001.
17. Spyropoulos, and C. S. Raghavendra, "Energy Efficient Communication in Ad Hoc Networks Using Directional Antennas," in Proc. IEEE INFOCOM '2002, June 2002.
18. Nasipuri, K. Li, and U. R. Sappidi, "Power Consumption and Throughput in Mobile Ad hoc Networks using Directional Antennas," in Proc. of 11th Conference on Computer Communications and Networks (ICCCN '02), October 2002.
- A. Balanis, Antenna Theory: Analysis and Design, 2nd ed. New York: Wiley, 1997.
19. IEEE Local and Metropolitan Area Network Standards Committee, Wireless LAN medium access control (MAC) and physical layer (PHY) specifications, IEEE standard 802.11-1999, 1999.
20. S. Bandyopadhyay, K. Hasuiki, S. Horisawa, and S. Tawara, "An Adaptive MAC and Directional Routing Protocol for Ad Hoc Wireless Network Using ESPAR Antenna," in Proc. ACM MobiHoc '2001.
21. <http://www.wolfram.com/products/mathematica/>
22. T. S. Rappaport, Wireless Communications: Principles and Practice, Prentice Hall, 1996
23. L. L. Xie, and P. R. Kumar, "A Network Information Theory for Wireless Communication: Scaling Laws and Optimal Operation," submitted to IEEE Transactions on Information Theory, April 2002.
24. Network connectivity paper