Fuzzy Dominance Matrix and its Application in Decision Making Problems

Sulekha Gope, Sujit Das

Abstract: This paper introduces the concept of fuzzy dominance matrix (FDM) for solving multiple attribute decision making (MADM) problems. During decision making process, dominance of one expert over others plays an important role to find out the optimal alternative(s). In uncertain decision making problems, often dominances are expressed using linguistic variables which can be represented by fuzzy dominance degree. We have proposed an algorithmic approach to solve multiple attribute decision making problems using FDM. Finally the proposed algorithm is illustrated using a numerical example.

Index Terms— fuzzy set, fuzzy dominance matrix, fuzzy decision matrix, multiple attribute decision making.

I. INTRODUCTION

Decision making is the study of identifying and choosing alternatives based on the values and preferences of the decision maker. Making a decision implies that there are alternative choices to be considered, and in such a case we want not only to identify as many of these alternatives as possible but to choose the one that best fits with our goals, objectives, desires, values, and so on [1]. Decision making is the most vital step in many real applications such as critical disease diagnosis, inventory planning, financial planning, risk assessment etc. Basically, decision making is the process for choosing the most appropriate one among a set of alternatives under provided criteria, attributes, objectives, or preferences [2]–[4]. As the situation demands, decision can be taken by a single decision maker or a group of decision makers. Actually, in complex situations, when numbers of alternatives, criteria, or objectives are more, then only a group of decision makers can provide better solutions. A number of researchers [5]-[15] have contributed to develop various decision making models to support human for making decisions under complex situations.

Aim of our paper is to introduce a new concept to represent a decision making system with the help of fuzzy dominance matrix. We have used fuzzy decision matrix to present the opinions of individual decision makers. This matrix is formed with a finite set of alternative and criteria, where opinion of a decision maker is presented using a fuzzy value. Fuzzy dominance matrix is used to find out the dominance degree of an expert over other expert on a set of alternative-attribute pair. This matrix uses positive symbol when 1st expert yields dominance over 2nd expert and negative symbol in the reverse case. To facilitate our discussion, some basic operations on fuzzy dominance matrix have been devised in this study.

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Then we have proposed an algorithmic approach to show the importance of fuzzy dominance matrix in decision making paradigm. Finally, the proposed algorithm has been validated using a numerical example.

The rest of this paper is organized as follows. Section II reviews some basic ideas related with this paper. In Section III, we propose the fuzzy dominance matrix and some basic operations on FDM. An algorithmic approach is proposed in section IV to present the application of FDM in decision making problems followed by a numerical example. Finally the key conclusions are given in section V.

II. PRELIMINARIES

This section briefly reviews the basic characteristics of fuzzy set and intuitionistic fuzzy set.

Let X be a universe of discourse, then a fuzzy set

$$A = \{< x, \mu_A(x) > | \ x \in X\}$$

defined by Zadeh [16] is characterized by a membership function $\mu_A : X \rightarrow [0,1]$, where $\mu_A(x)$ represents the degree of membership of the element x to the set A . Atanassov [17]-[18] introduced a more general version fuzzy set called intuitionistic fuzzy set, shown as follows:

$$\hat{A} = \{< x, \mu_{\hat{A}}(x), \nu_{\hat{A}}(x) > | \ x \in X\}$$

which is characterized by a membership function

$$\mu_{\hat{A}} : X \rightarrow [0,1]$$

and a non-membership function

$$\nu_{\hat{A}} : X \rightarrow [0,1]$$

with the condition

$$0 \le \mu_{\hat{A}}(x) + \ \nu_{\hat{A}}(x) \le 1$$

for all $x \in X$, where the numbers $\mu_{\hat{A}}(x)$ and $\nu_{\hat{A}}(x)$ denotes the degree of membership and the degree of non membership respectively of the element *x* to the set $\hat{A}$. For each intuitionistic fuzzy set $\hat{A}$ in *X*, the hesitation margin or intuitionistic fuzzy index which expresses a lack of knowledge of whether *x* belongs to *A* or not is defined as

$$\pi_{\hat{A}}(x) = 1 - \{\mu_{\hat{A}}(x) + \nu_{\hat{A}}(x)\}$$

where $x \in X$ and it is hesitation degree of whether $x \in \hat{A}$ or $x \notin \hat{A}$. Also for each $x \in X$,

$$0 \le \pi_{\hat{A}}(x) \le 1.$$

Note that for an IFS $\hat{A}$, if $\mu_{\hat{A}}(x) = 0$ then

$\nu_{\hat{A}}(x)+\pi_{\hat{A}}(x)=1$ and if $\mu_{\hat{A}}(x)=1$ then $\nu_{\hat{A}}(x)=0$ and $\pi_{\hat{A}}(x)=0$. For each fuzzy set A in X we can say $\pi_A(x)=1-\mu_A(x)-[1-\mu_A(x)]=0,$ for every $x\in X$.

III. FUZZY DOMINANCE MATRIX

The problem we deal with is the choosing of best alternative(s) among a finite set of alternatives $X=\{x_1,x_2,...,x_m\}, m\geq 2,$ depending on a finite set of attributes $C=\{c_1,c_2,...,c_n\}, n\geq 2$. The alternatives will be classified from best to worst, using the information known (attributes) according to a set of experts $E=\{e_1,e_2,...,e_k\}, k\geq 2.$ Fuzzy dominance degree represents the dominance of an expert over other expert on a alternative-attribute pair.

A fuzzy dominance matrix R on a set of alternatives *X* is a fuzzy set on the product set $E\times E$. It is characterized by a membership function $\mu_R: E\times E\rightarrow[0,1].$

When cardinality of *X* is small, the dominance matrix may be conveniently represented by $n\times n$ matrix, $R=\left(r_{ij}\right)$, being $r_{ij}=\mu_R(e_i,e_j), \forall i,j\in\{1,2,...,k\}, i\neq j,$ interpreted as the dominance degree or intensity of the expert e_i over e_j on the set of $(x_i,c_j), i\in\{1,2,...,m\}, j\in\{1,2,...,n\}$ where $r_{ij}=0$ indicates indifference between e_i and e_j $(e_i\sim e_j)$, $r_{ij}>0$ indicates that e_i is preferred to e_j $(e_i\succ e_j)$, $r_{ij}<0$ indicates that e_j is preferred to e_i $(e_i\prec e_j)$. Dominance degree of expert e_i over e_j on the set of $(x_i,c_j), i\in\{1,2,...,m\}, j\in\{1,2,...,n\}$ can be calculated as

$r_{ij}^{A,B}=\left(d_{ij}^A-d_{ij}^B\right), 1\leq i\leq m, 1\leq j\leq n, A\in E, B\in E,$

where d_{ij}^A and d_{ij}^B are fuzzy decision matrices of experts *A* and *B* respectively.

Addition of fuzzy dominance matrix

Two fuzzy dominance matrix r_{ij}^A and r_{ij}^B where $(A,B)\in E$ are said to be conformable for addition, when two matrices are of same order, i.e., number of rows and columns are same. The addition of two fuzzy dominance matrixes r_{ij}^A and r_{ij}^B of order $n\times n$ is defined by

$$R=R^A\oplus R^B=\left(\max\left\{r_{ij}^A,r_{ij}^B\right\}\right)(\forall i,j\in\{1,2,...,n\}),$$

where R is also an *n x n* fuzzy dominance matrix.
Let,

$$R^A=\left(r_{ij}^A\right)=\begin{bmatrix}0.8&0.7&0.5&0.9\\0.4&0.3&0.8&0.4\\0.6&0.4&0.2&0.6\\0.7&0.5&0.5&0.7\end{bmatrix},$$

$$R^B=\left(r_{ij}^B\right)=\begin{bmatrix}0.7&0.6&0.6&0.8\\0.3&0.3&0.4&0.4\\0.5&0.5&0.7&0.6\\0.6&0.4&0.9&0.6\end{bmatrix}.$$

Then the resultant FDM *R* will be as below.

$$R=R^A\oplus R^B=\begin{bmatrix}0.8&0.7&0.6&0.9\\0.4&0.3&0.8&0.4\\0.6&0.5&0.7&0.6\\0.7&0.5&0.9&0.7\end{bmatrix}.$$

Subtraction of fuzzy dominance matrix

Two fuzzy dominance matrix r_{ij}^A and r_{ij}^B where $(A,B)\in E$ are said to be conformable for subtraction when two matrices are of same order, i.e., number of rows and columns are same. The subtraction of two fuzzy dominance matrixes r_{ij}^A and r_{ij}^B of order $n\times n$ is defined by

$$R=R^A-R^B=\left(\min\left\{r_{ij}^A,r_{ij}^B\right\}\right)(\forall i,j\in\{1,2,...,n\})$$

where R is also an *n x n* fuzzy dominance matrix.
Let,

$$R^A=\left(r_{ij}^A\right)=\begin{bmatrix}0.8&0.7&0.5&0.9\\0.4&0.3&0.8&0.4\\0.6&0.4&0.2&0.6\\0.7&0.5&0.5&0.7\end{bmatrix},$$

$$R^B=\left(r_{ij}^B\right)=\begin{bmatrix}0.7&0.6&0.6&0.8\\0.3&0.3&0.4&0.4\\0.5&0.5&0.7&0.6\\0.6&0.4&0.9&0.6\end{bmatrix},$$

Then the resultant FDM *R* will be as follows:

$$R=R^A-R^B=\begin{bmatrix}0.7&0.6&0.5&0.8\\0.3&0.3&0.4&0.4\\0.5&0.4&0.2&0.6\\0.6&0.4&0.5&0.6\end{bmatrix}.$$

Product of fuzzy dominance matrix

Two fuzzy dominance matrix r_{ij}^A and r_{ij}^B where $(A,B)\in E$ are said to be conformable for multiplication operation when two matrices are of same order, i.e., number of rows and columns are same. The multiplication of two fuzzy dominance matrices r_{ij}^A and r_{ij}^B of order $n\times n$ is defined by

$$R=R^A\otimes R^B=\left(Max_{j=1}^n(Max\left\{r_{ij}^A,r_{ij}^B\right\})\right)$$
$$(\forall i,j\in\{i=1,2,...,n\}),$$

where *R* is also an *n x n* fuzzy dominance matrix.

Let,

$$R^A = \left(r_{ij}^A\right) = \begin{bmatrix} 0.8 & 0.7 & 0.5 & 0.9 \\ 0.4 & 0.3 & 0.8 & 0.4 \\ 0.6 & 0.4 & 0.2 & 0.6 \\ 0.7 & 0.5 & 0.5 & 0.7 \end{bmatrix},$$

$$R^B = \left(r_{ij}^B\right) = \begin{bmatrix} 0.7 & 0.6 & 0.6 & 0.8 \\ 0.3 & 0.3 & 0.4 & 0.4 \\ 0.5 & 0.5 & 0.7 & 0.6 \\ 0.6 & 0.4 & 0.9 & 0.6 \end{bmatrix}.$$

Then the resultant FDM *R* will be as below.

$$R = R^A \otimes R^B = \begin{bmatrix} 0.9 & 0.9 & 0.9 & 0.9 \\ 0.7 & 0.8 & 0.9 & 0.8 \\ 0.7 & 0.6 & 0.9 & 0.8 \\ 0.7 & 0.7 & 0.9 & 0.8 \end{bmatrix}.$$

IV. ALGORITHMIC APPROACH & ANALYSIS

Firstly, we propose an algorithm for multiple attribute decision making using the opinions of two experts. Next, we extend the algorithm for three or more number of experts.

Algorithm 1: Decision making procedure using two experts

Step1: Fuzzy decision matrices of two expert e_1 and e_2 are constructed and taken as input. For a multiple attribute decision making problem with *m* alternatives $X = \{x_1, x_2, ..., x_m\}, m \geq 2$ and *n* attributes $C = \{c_1, c_2, ..., c_n\}, n \geq 2$, a fuzzy decision matrix $D = \left(d_{ij}\right)$ can be represented as below.

$$D = \left(d_{ij}\right)_{m\times n} = \begin{pmatrix} d_{11} & d_{12} & \cdots & d_{1n} \\ d_{21} & d_{22} & \cdots & d_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ d_{m1} & d_{m2} & \cdots & d_{mn} \end{pmatrix}, \; d_{ij} \in [0,1].$$

Step 2: Fuzzy dominance matrix *R* is constructed based on the subtraction of fuzzy decision matrices *D* of individual experts.

$$R = \left(r_{ij}\right)_{m\times n} = \begin{pmatrix} r_{11} & r_{12} & \cdots & r_{1n} \\ r_{21} & r_{22} & \cdots & r_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ r_{m1} & r_{m2} & \cdots & r_{mn} \end{pmatrix}, \; r_{ij} \in [-1,1],$$

$$r_{ij}^{A,B} = \left(d_{ij}^A - d_{ij}^B\right), 1 \leq i \leq m, 1 \leq j \leq n, A \in E, B \in E.$$

Step3: Choice value Ch^i of the i^{th} alternative is calculated by adding all dominance values corresponding to that alternative.

$$Ch^i = \sum_{j=1}^{n} (r_{ij}), i \in [1, 2, ..., m].$$

Step 4: If $Ch^k = \max_i Ch^i, \forall i = [1, 2, ..., m]$, alternative x_k is selected.
Step 5: If *k* has more than one value, then any one of x_k may be chosen.

Algorithm 2: Decision making procedure using three or more experts

Step 1: Fuzzy decision matrices of a set of experts $E = \{e_1, e_2, ..., e_k\}, k \geq 2$ are taken as input.

Step 2: Fuzzy dominance matrices $R^i, i > 0$ are constructed based on the subtraction of fuzzy decision matrices $D^j, j = [1, 2, ..., k]$ of individual experts.

$$R^i = D^j - D^l, j = [1, 2, ..., k], l = j + \delta, \delta > 0.$$

Step 3: Fuzzy dominance matrices $R^i, i > 0$ are aggregated by the expression given below.

$R = \max_{l=1}^{p} (r_{ij}^l), i = [1, 2, ..., m], j = [1, 2, ..., n]$, *p* is the number of fuzzy dominance matrices.
Step 4: In aggregated fuzzy dominance relations, Choice value Ch^i of the i^{th} alternative is calculated by adding all dominance values corresponding to that alternative.

$$Ch^i = \sum_{j=1}^{n} (r_{ij}), i \in [1, 2, ..., m].$$

Step 5: If $Ch^k = \max_i Ch^i, \forall i = [1, 2, ..., m]$, alternative x_k is selected.
Step 6: If *k* has more than one value, then any one of x_k may be chosen.

Example 1: Let $U = \{M_1, M_2, M_3, M_4\}$ be a set of four candidates and P = {Highest Qualification, Knowledge, Previous Experience, Hard Worker} be the set of parameters (attributes), given by, $P = \{p_1, p_2, p_3, p_4\}$. A set of three experts $E = \{e_1, e_2, e_3\}$ want to evaluate the best candidate as per their knowledgebase. Fuzzy decision matrices of expert e_1, e_2 and e_3 are given in table I, table II, and table III respectively. Fuzzy dominance matrices are calculated in table IV, table V, and table VI where table IV shows fuzzy dominance relation of expert e_1 and e_2, table V shows fuzzy dominance relation of expert e_1 and e_3, and table VI shows fuzzy dominance relation of expert e_2 and e_3. These fuzzy dominance relations are aggregated in table VII where the choice values of various alternatives are calculated. In this study, since the maximum choice value is 1.3, so candidate M_1 will be selected.

Table I. Opinion of e_1

	p_1	p_2	p_3	p_4
M_1	0.8	0.7	0.5	0.9
M_2	0.4	0.3	0.8	0.4
M_3	0.6	0.4	0.2	0.6
M_4	0.7	0.5	0.5	0.7

Table II. Opinion of e_2

	p_1	p_2	p_3	p_4
M_1	0.7	0.6	0.6	0.8

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M_2	0.3	0.3	0.4	0.4
M_3	0.5	0.5	0.7	0.6
M_4	0.6	0.4	0.9	0.6

Table III. Opinion of e_3

	p_1	p_2	p_3	p_4
M_1	0.1	0.8	0.2	0.9
M_2	0.8	0.3	0.9	0.4
M_3	0.3	0.5	0.6	0.7
M_4	0.5	0.6	0.4	0.6

Table IV. FDM 1

	p_1	p_2	p_3	p_4
M_1	0.1	0.1	-0.1	0.1
M_2	0.1	0	0.4	0
M_3	0.1	-0.1	-0.5	0
M_4	0.1	0.1	-0.4	0.1

Table V. FDM 2

	p_1	p_2	p_3	p_4
M_1	0.7	-0.1	0.3	0
M_2	-0.4	0	-0.1	0
M_3	0.3	-0.1	-0.4	-0.1
M_4	0.2	-0.1	0.1	0.1

Table VI. FDM 3

	p_1	p_2	p_3	p_4
M_1	0.6	-0.2	0.4	-0.1
M_2	-0.5	0	-0.5	0
M_3	0.2	0	0.1	-0.1
M_4	0.1	-0.2	0.5	0

Table VII. Aggregated FDM

	p_1	p_2	p_3	p_4	**Choice value**
M_1	0.7	0.1	0.4	0.1	**1.3**
M_2	0.1	0	0.4	0	0.5
M_3	0.3	0	0.1	0	0.4
M_4	0.2	0.1	0.5	0.1	0.9

V. CONCLUSIONS

This study has introduced fuzzy dominance matrix for solving multiple attribute decision making problems in uncertain environment. FDM is mainly useful in such situations where decision makers are able to express their opinions about all the attributes in terms of fuzzy value. In simple way, when there is no missing or unknown information, FDM is proved to be more effective. This paper has used fuzzy decision matrix to present the opinions of decision makers. FDMs are constructed by using the fuzzy decision matrices of individual decision makers. In this work, we have proposed two algorithms: algorithm 1 is applicable for two decision makers and algorithm 2 is applicable for three or more decision makers. In future, researchers can develop more efficient decision making algorithm using fuzzy dominance matrix.

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