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Design of Programmable Linear Phase Equiripple FIR Filter based Chebyshev and Remez Algorithms

Abdulhassan Nasayif Aldujaili

Abstract*— *In this paper, a novel algorithm of programmable linear phase Equiripple finite impulse response (PFIR) filter is developed and improved. The proposed algorithm is incorporated with Remez algorithm to calculate the minimum filter order and Chebyshev algorithm to minimize the error by optimize the filter order. A new algorithm and technique has been used to reduce the ripple in the pass-band filter response by insert different weights used in the different band. Additionally, the weights at the pass-band region are set to 30 times more than the stop-band weights to improve the adjacent band rejection and blocker response. Results show an development of passband ripple with improvement in the adjacent band rejection of 18% and 11% in blocker requirements more than conventional filter. These results confirm the validity of the proposed algorithms and the techniques used are promising to support the new generation requirements of wireless communication system.

***Index Terms*— Linear phase, Equiripple, FIR, Chebyshev, Remez**

I. INTRODUCTION

The Chebyshev algorithm or well known a Mini-max algorithm is useful due to it permits the designer to openly specify band edge and relative error sizes in each band. The FIR filters that minimize a Chebyshev error criterion can be found with Remez algorithm. Together these methods are iterative numerical algorithms which could be used for extremely general functions D(w) and W (w) while many implementation design simply for piecewise linear functions. The Remez algorithm is not as general as the linear programming advance, but it's extremely robust, converges very fast to the best solution and is extensively used [1]. To design all four type of linear phase FIR filter (I, II, III, IV), the Remez algorithm could be used properly. The Chebyshev approximation looking for the polynomial of degree (n) that approximates the specified function in the specified interval such that the absolute maximum error is minimized. The error here is defined as the difference between the function and the polynomial. Chebyshev proved that such polynomial exists and it is single. He also gave the criteria for a polynomial to be a minimax polynomial [2].

II. CHEBYSHEV AND REMEZ ALGORITHMS

Assuming that the given interval is [a, b] Chebyshev criteria states that if Pn (X) is the minimax polynomial of degree n then there must be at least (n+2) points in this interval at which the error function attains the absolute maximum value with alternating sign as shown in figure 1 for n = 3 and by the following equations:

$$a \le x_0 < x_1 < \cdots < x_{n+1} \le b$$

$$F(x_i) - P_n(x_i) = (-1)^i E\,, \qquad i = 0,1,\dots n+1 \qquad (1)$$

$$E = \pm\, max_{a \le x \le b}\, |F(x) - P_n(x)| \qquad (2)$$

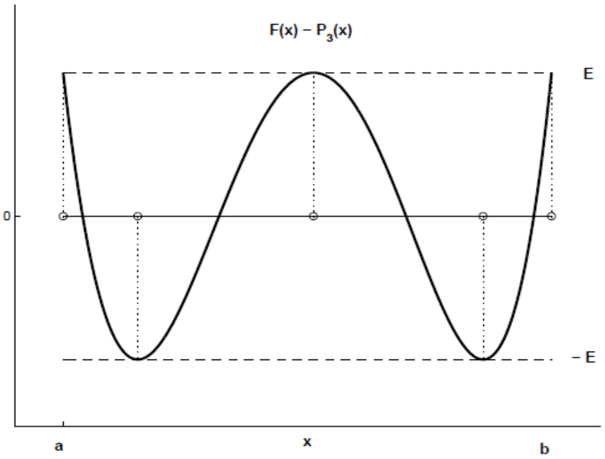

Figure 1: The Chebyshev Criteria of Third Order Polynomial

The minimax polynomial can be computed analytically up to n = 1. For higher order a numerical method due to Remez [2] has to be employed. Remez algorithm is an iterative algorithm. We start the first iteration by an arbitrary set of (n + 2) points in the given interval. Each iteration is composed of two steps. In the first step we compute the coefficients such that the error function takes equal magnitude with alternating sign at (n+2) given points.

$$F(x_i) - P_n(x_i) = (-1)^i E \qquad (3)$$

$$F(x_i) - [c_0 + c_1(x_i - a) + c_2(x_i - a)^2 + \cdots + c_n(x_i - a)^n] = (-1)^i E \qquad (4)$$

$$c_0 + c_1 h_i + \cdots + c_n h_i^{\,i} + (-1)^i E = F(x_i), \qquad i = 0,1,2,\dots n+1 \qquad (5)$$

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Dr. Abdulhassan Nasayif Aldujaili, Department of Computer, Diglah University College, Baghdad, Iraq.

The above equations are proved to be independent [2], therefore one can resolve them using some method from linear algebra to obtain the values of the coefficients and the error at the specified (n + 2) points. Following the first step one can compute the coefficients such that the error function at the given (n + 2) points is equivalent in magnitude and irregular in sign. Though, the magnitude of this error is not the absolute maximum magnitude in the given interval [a, b]. Consequently the minimax form is still not meet. However, new set of points may give better solution. Remez algorithms looking for new set of (n + 2) points which will attain the minimax condition and this approach is called the exchange approach. In this case, two exchange techniques will be used:

1. The first is to change a single point in the present set of $(n + 2)$ points to get a new set of points
2. The second is to exchange all points in the present set of $(n + 2)$ points to get a new set of points.

In the second exchange, the error will alternates in sign at the $(n+2)$ points of the first step. The new set of $(n + 2)$ points could be used in the first step of the following iteration and one can repeat the two steps many time until the difference between the old $(n+2)$ points and the new $(n+2)$ points lies under a given threshold. The second step could be represented graphically as shown in Figure 2 for third order polynomial.

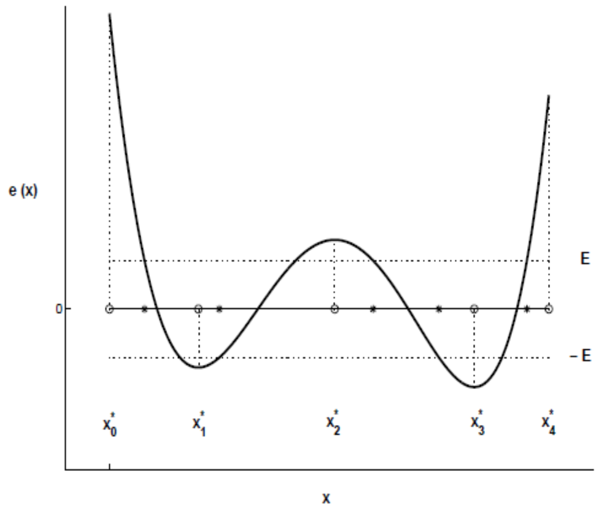

Figure 2: The Second Step of Remez Algorithm

The magnitude of the error function at the final set of the (n+ 2) points (E) represents the maximum absolute value of the approximation error. Hence, the error function E(ω) and a weight function W(ω) which define the relative significance of the error at any certain frequency ω. Then, the error function can be described as in [3].[4],[5],[6]

$$E(\omega) = W(\omega)[A_d(\omega) - A(\omega)] \tag{6}$$

Where A_d (ω) is the desired amplitude response and A(ω) is the designed amplitude response. A simple weight function W(ω), can be defined in ([5] as:

$$W(\omega) = \begin{cases} 1, \omega \in (passband) \\ 0, \omega \in (stopband) \end{cases} \tag{7}$$

And the actual amplitude response of a linear-phase FIR filter could be represented as a product of two filters functions as given by [5]

$$A(\omega) = F(\omega)\, G(\omega) \tag{8}$$

and

$$G(\omega) = \sum_{k=0}^{M} g[k](\cos\omega)^k, \quad M = \frac{N}{2}, \quad K = 0,1,2 \dots\dots \tag{9}$$

where *F(ω)* and M are obtained from Table (1).

Table 1: Parameters of the Four FIR Filters [7], [5]

Type	I	II	III	IV
Order	Even	Odd	Even	Odd
F(ω)	1	Cos(ω/2)	Sin(ω)	Sin(ω/2)
M	N/2	(N-1)/2	(N-2)/2	(N-1)/2
ω_0	0	0	$\pi/2$	$\pi/2$

The filter coefficients $g[k]$ should be obtained to reduce the maximum absolute weighted error |E(ω)|. A specific algorithm should be used to reduce the magnitude of the error |E(ω)| across the specified pass-band and stop-band. Thus, to optimize the error in pass-band response, the mini-max algorithm could be used as mentioned in [3],[4]

$$\|E(\omega)\|_\infty = max_{\omega\in[0,\pi]}|W(\omega)[A_d(\omega) - A(\omega)]| \tag{10}$$

where ω is in the desired operating frequency range of the filter. The solution to this problem is called the best weighted Chebyshev approximation since it minimizes the maximum value of the error, it is also called the minimax solution. The Remez algorithm for computing the best Chebyshev solution uses the alternation theorem. This theorem characterizes the best Chebyshev solution.

III. EQUIRIPPLE FIR FILTER DESIGN

The design of linear-phase FIR filters depend on the Chebyshev error criteria or the so called mini-max algorithms and Remez algorithms is present in this section. The mini max algorithms are helpful since it permits to openly specify the band-edge and virtual error sizes in every band. The linear-phase FIR filters that reduce the error could be found with Remez algorithm. The Remez algorithms is an reiterate arithmetical algorithms and could be used for general functions of A(ω) and W(ω). The Remez algorithms could be used in designing all types of linear-phase filters (I, II, III, and IV). The weighted error function is known in Equation 6 and the response of type-I FIR filter is known by Equation 9. Then the problem formulation can be defined as Given M is the filter length, the desired (real value) amplitude function is $A(\omega)$ and the weighting function is W(ω). To discover the linear-phase filter that reduce the weighted Chebyshev error, the mini max algorithms should be used as given in Equation 10 Sven, (2004) and McCLellan, (1975). The weighted Chebyshev approximation to $A(\omega)$ could be considered is the best solution of this problem. Due to minimize the maximum value of the error, it is called the mini max solution.

The design of FIR filter by the Remez algorithm, it is of consideration to know an estimate of the filter order N. The filter order is known by N= M – 1, and the filter length M may be estimated by means of Kaisers estimate method recognized by Equation 11. First, the order N of the desired filter can be found by using the formula proposed by [4],[5],[6] and [7]

$$M = \frac{-20\log_{10}\left(\sqrt{\delta_p\delta_s}\right)-13}{14.6\ \Delta f} + 1, \quad \Delta f = \frac{\omega_s-\omega_p}{2\pi} \tag{11}$$

where ω_p is the pass-band-edge digital frequency, ω_s is the stop-band-edge digital frequency, δ p is the pass-band allowed deviation, δs is the stop-band allowed deviation, and

$$\delta_p = \frac{10^{A_P/20}-1}{10^{A_P/20}+1} \quad A_p = -20\log_{10}\frac{1-\delta_p}{1+\delta_p} > 0(\cong 0) \tag{12}$$

$$\delta_s = 10^{A_s/20}\ , \quad A_s = -20\log_{10}\frac{\delta_s}{1+\delta_p} > 0\ (\geq 1) \tag{13}$$

where A_p and A_s are the attenuations on the pass-band and stop-band respectively. From Equations 6,9,11, it can be concluded that, when the transition width Δf decreases, the filter length M increase (Equation 11), so if the coefficients g [k] in Equation 9 is increased, the maximum absolute weighted error |E(ω)| in Equation 3.15 must be decrease due to high amplitude response A(ω) of the real filter. If filter type (I) is used for example, then from Table 1, F (ω)=1, and, M=N/2, therefore, the designed amplitude response in Equation 8 becomes A(ω) = G (ω□) and the filter coefficients $g[k]$ becomes:

$$g[k] = h[k] = \begin{cases} h\left[\frac{N}{2}\right] & K = 0 \\ 2h\left[\frac{N}{2}-K\right] & 1 \leq K \leq N/2 \end{cases} \tag{14}$$

where N is the filter order
Substituting Equation 14 in Equation 9 yields

$$A(\omega) = G(\omega) = h\left[\frac{N}{2}\right] + \sum_{K=1}^{M} 2h\left[\frac{N}{2}-k\right](\cos\omega)^k \tag{15}$$

Where M = N/2
Then the problem could be stated as

$$|E(\omega)| = \max_{\omega\in\varphi} W(\omega)\ \left|A_d(\omega) - \left(h\left[\frac{N}{2}\right] + \sum_{K=1}^{M} 2h\left[\frac{N}{2}-k\right](\cos\omega)^k\right)\right| \tag{16}$$

where φ is the required frequency in period (0-π)
This problem is a mini max problem (i.e., the error could be optimized by finding the minimum filter coefficients $g[k]$ that reduce the maximum absolute weighted error). The best solution to this problem lies in finding the special algorithms to guess the length of the filter with less error as possible. This problem could be summarized in Figure 3.

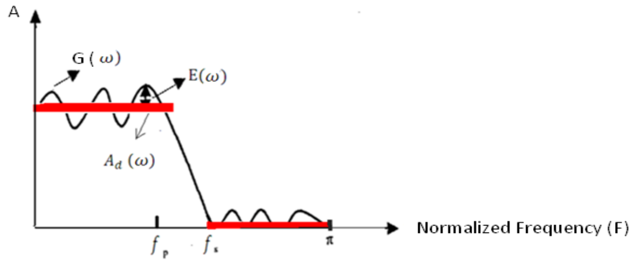

Figure 3: The Error between Actual Response and the Desired Response

In several applications a linear-phase filter is attractive, and as the most attractive features of finite impulse response filter, it could be designed in an easy way to get linearity, and could be symmetric or non symmetric. This type of filter response, can be designed it easily, and the problem becomes an arithmetic problem. The problem is to get the filter that minimizes the maximum error between the real and ideal filters. Equiripple filter that has a linear phase can be used because maximum ripple in its response is least compared with other filters in the same order. This filter has the peak deviation close to ideal filter and, the problem here is to discover the filter that decreases the power of the error between the real and ideal filter. A weighted-Chebyshev method could be used to design this filter because its design is best, and the method is extremely flexible, it yields Equiriple solution, and minimum filter order will get for the required specifications and implies a more capable and faster filter for real-time application.

The M-file program illustrates bellow shows the idea of inserting different weights in the filter response. Because of the different weights, the resulting FIR filter response is given by fvtool in MATLAB [9] shows different stop-band attenuation values. Additionally, the weights at the pass band have been determined to be 30 times more than the weight of the stop-band.

```
N = 64;
Fs = 541666;
F = [0 80e3 100e3 100e3 108e3 Fs/2]/(Fs/2);
A = [1 1 0 0 0  0];
W = [30 1 30];  % Weight the passband 30 times more than the stopband
pfir = firgr(N,F,A,W);
pfirq = fi(pfir,true,16);
Hpfir = mfilt.firdecim(2,double(pfirq));
 fvtool (Hpfir)
```

The weights were determined to improve the adjacent band rejections. Results show adjacent band rejection improvement of 18% more than conventional filter, as shown in Figure 4. The two stopband in proposed filter is highlight in Figure 5 and the passband ripple is 0,02dB as illustrated in Figure 6

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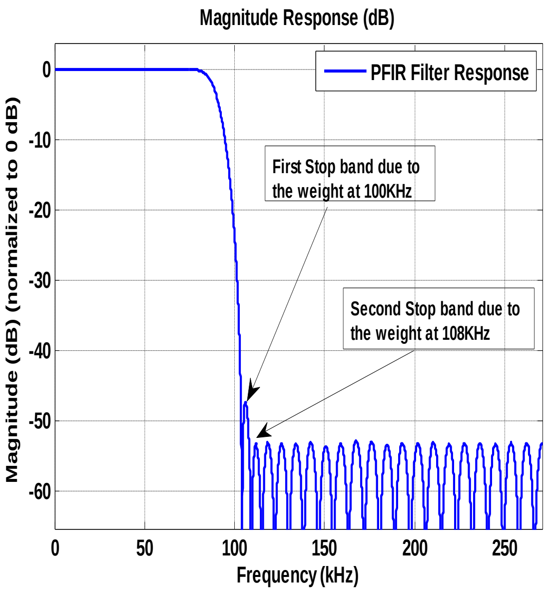

Figure 4: Finite Impulse Response (FIR) Response with Different Stop-Band

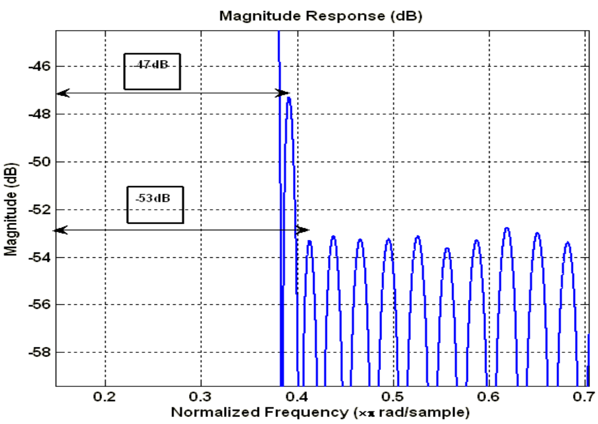

Figure 5: The Two Stopband Rejion at 100 kHz and 80 kHz

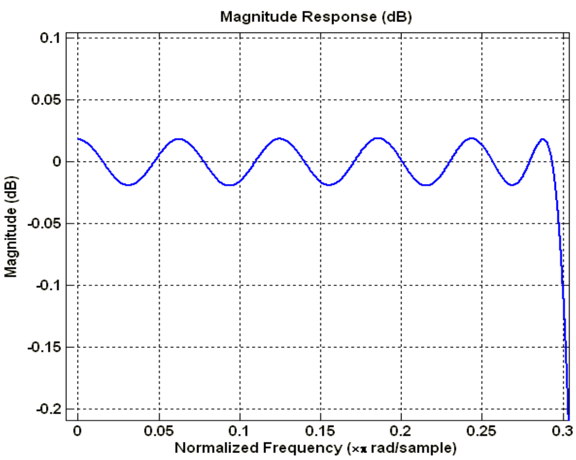

Figure 6: Pass Band Ripple of Proposed Filter

IV. Conclusion

In this paper, the tutorial guideline of design the efficient linear phase Equiripple FIR filter depend on Chebyshev and Remez algorithms which minimize the error rate in the filter response. The use of different weight at different band in the filter response and the accurate choice of its position lead to compensate the adjacent rejection and yield Equiripple FIR filter. The efficiency and robustness of Remez algorithm lead to increase the rejection scheme, however, the Remez algorithm continues to be the method of choice for the design of linear-phase filters, multi band filters, differentiators, Hilbert transformers. Regardless of the improvements described, the Remez continues to require a large amount of computation and further research needs to be undertaken to make it suitable for applications where the design has to be done in real or quasi-real time. Extra potential require to be done on the design of systems with complex coefficients, on the design of approximately linear-phase filters, on the application of the Remez algorithm for the design of recursive (IIR) filters, and also to reduce the amount of computation additional.

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Abdulhassa Aldujaili, Received His Bsc. degree in the computer engineering from University of Technology Baghdad – Iraq-1972 and Msc from UCNW_UK at 1981. Doctor of philosophy in computer engineering from UCNW_UK -1984. He is currently Debuty Dean of Dijlah University College, Baghdad-Iraq a Ministry of high education and scientific research-Iraq. He has been a full-time lecturer in the Computer Department , Baghdad, Iraq, since March 2005. He also worked as senior researcher in the Iraqi Center of Development and Research since 1994.