Particle Swarm Optimization Approach to Harmonic Reduction in Voltage Source Multilevel Inverter

Adeyemo I. A, Okediran, O. O, Oyeleye, C. A.

Abstract – In Selective Harmonic Elimination-Pulse Width Modulation (SHE-PWM) technique, optimal switching angles at fundamental switching frequency are computed such that low order harmonics are eliminated, while the fundamental voltage is obtained as desired. The main challenge associated with SHE-PWM technique is that a specified number of transcendental nonlinear equations known as Selective Harmonic Elimination (SHE) equations have to be solved to obtain the appropriate switching angles. In this paper, Particle Swarm Optimization (PSO) algorithm with random initial values is proposed for solving SHE equations of an 11-level inverter. The proposed method is derivative-free, accurate and globally convergent. Both computational and MATLAB simulation results show that the proposed method is highly efficient for elimination of the selected low order harmonics as well as minimization of the total harmonic distortion (THD).

Keywords - Multilevel inverter, PSO, modulation index, and harmonics.

I. INTRODUCTION

A multilevel voltage source inverter is a power electronic system that synthesizes a near sinusoidal output voltage from several DC voltages using pulse width modulation technique. Due to the smaller voltage steps in the output staircase waveform, multilevel inverter has many advantages over the traditional two-level inverter among which are: improved power quality, low switching losses, lower dv/dt stresses on the load, lower electro-magnetic interference (EMI) and ability to attain a higher voltage without transformer [1]. The use of multilevel inverter is prevalent in high power industrial applications such as drives, Flexible AC Transmission Systems (FACTS), Hybrid Electric Vehicle (HEV), High Voltage Direct Current (HVDC) lines, and utility. Several pulse width modulation techniques used in conventional two-level inverter have been modified and deployed in multilevel inverters. These include Sinusoidal Pulse Width Modulation (SPWM), Selective Harmonic Elimination (SHE) method, Space Vector Control (SVC), and Space Vector Pulse Width Modulation (SVPWM) [2], [3]. Selective Harmonics Elimination-Pulse Width Modulation (SHE-PWM) technique at fundamental switching frequency however, arguably gives the best result because of its high spectral performance and considerably reduced switching loss.

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The objective of SHE-PWM technique is to eliminate low order harmonics that are more harmful and difficult to remove with filter while the fundamental voltage is obtained at a desired value. The main challenge associated with the SHE-PWM technique is that it involves the solution of a specified number of transcendental nonlinear equations known as SHE problem that gives the relation between the optimal switching angles and the harmonic components at different modulation indices. SHE problem belongs to the class of continuous combinatorial optimization problems known as NP-complete. Several methods that have been reported for solving SHE problem can be classified into two groups: The first group is based on deterministic approach using exact algorithms. Newton Raphson iterative method [4] is one of these algorithms. The main disadvantage of iterative methods is that they diverge if the arbitrarily chosen initial values are not sufficiently close to the roots. They also risk being trapped at local optima and fail to give all the possible solution sets. The theory of symmetric polynomials and resultants [5] has been proposed to determine the solutions of the SHE equations. A difficulty with this approach is that as the number of levels increases, the order of the polynomials becomes very high, thereby making the computations of solutions of these polynomial equations very complex. Another approach uses Walsh functions [6], [7], [8] where solving linear equations, instead of non-linear transcendental equations, optimizes the switching angle. The method results in a set of algebraic matrix equations and the calculation of the optimal switching angles is a complex and time-consuming operation. The second group is based on probabilistic approach using heuristics that minimize rather than eliminate the selected harmonics. Population-based Evolutionary Algorithms (EAs) such as Genetic Algorithm [9], Particle Swarm Optimization [10], Ant Colony System [11] and Bee Algorithm [12] have been reported for computing the switching angles that eliminate 5^{th} and 7^{th} harmonics in 7-level inverter. The main benefits of EAs are improved convergence and the ability to find multiple solution sets over a wide range of modulation indices. These can be attributed to the parallel nature of EAs i.e. a search through a population of solutions rather than a sequential search for individual solutions, as in iterative method. EAs are derivative free and are successful in locating the optimal solution, but they are usually slow in convergence and require much computing time.

II. MULTILEVEL INVERTER

A. *Multilevel Inverter Topologies*

The concept of multilevel inverters was developed from the idea of step approximation of sinusoid [13]. Basically, there are three main multilevel topologies. These are Diode-Clamped Multilevel Inverter [14], Capacitor-Clamped Multilevel Inverter [15], and Cascaded H-bridge Multilevel Inverter with separate DC sources [16]. Among the topologies, cascaded H-bridge inverter requires the least number of components, and its modular structure as well as circuit layout flexibility makes it suitable for high voltage and high power applications.

Cascaded H-bridge multilevel inverter is formed by connecting several single-phase H-bridge inverters in series as shown in Fig.1 for an 11-level inverter. The number of output voltage levels in a cascaded H-Bridge inverter is given by N =2S +1, where S is the number of H-bridges per phase connected in cascade. By different combinations of the four switches S_1, S_2, S_3, and S_4 shown in the Fig.1, each H-bridge switch can generate a square wave voltage waveform with different duty cycle on the AC side. To obtain $+V_{dc}$, switches S_1 and S_4 are turned on, whereas $-V_{dc}$ can be obtained by turning on switches S_2 and S_3. By turning on S_1 and S_2, or S_3 and S_4, the output voltage is zero.

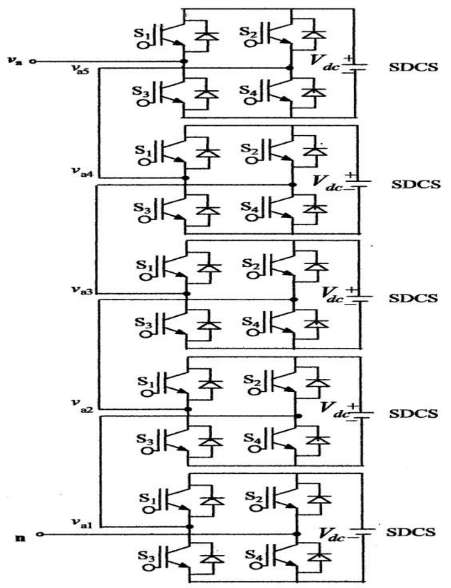

Fig.1. Single-phase structure of an 11-level cascaded H-bridge multilevel converter

The outputs of H-bridge switches are connected in series such that the synthesized AC voltage waveform is the summation of all voltages from the cascaded H-bridge cells [4], [5].

B. *MATHEMATICAL MODEL OF SHE-PWM*

Generally, any periodic waveform such as the staircase waveform shown in Fig. 2 can be shown to be the superposition of a fundamental signal and a set of harmonic components. By applying Fourier transformation, these components can be extracted since the frequency of each harmonic component is an integral multiple of its fundamental [17].

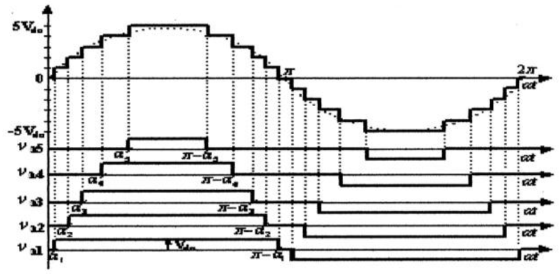

Fig. 2. Output voltage waveform of an 11-level inverter.

Assuming quarter-wave symmetry and the equal amplitude of all DC sources, the Fourier series expansion of the staircase output voltage waveform shown in Fig. 2 is given by equation (1).

$$V(\omega t) = V_n(\alpha)\sin(n\omega t) \quad (1)$$

Where

$$V_n(\alpha) = \frac{4V_{dc}}{n\pi}\sum_{k=1}^{S}\cos(n\alpha_k)\text{, for odd n} \quad (2)$$

$$V_n(\alpha) = 0\text{, for even n} \quad (3)$$

In three-phase power system, the triplen harmonics in each phase need not be cancelled as they automatically cancel in the line-to-line voltages as a result only non-triplen odd harmonics are present in the line-to-line voltages [4]

Combining equations (1), (2) and (3),

$$v(\omega t) = \sum_{n=1,3,5...}^{\infty}\frac{4V_{dc}}{n\pi}(\cos(n\alpha_1) + \cos(n\alpha_2) + ... + \cos(n\alpha_S))\sin n\omega t) \quad (4)$$

Subject to $0 < \alpha_1 < \alpha_2 < ..\alpha_S \le \pi/2$

Where, S is the number of switching angles and n is the harmonic order. Generally, for S number of switching angles, one switching angle is used for the desired fundamental output voltage V_1 and the remaining (S-1) switching angles are used to eliminate certain low order harmonics that dominate the Total Harmonic Distortion (THD) such that equation (4) becomes

$$V(\omega t) = V_1\sin(\omega t) \quad (5)$$

From equation (4), the expression for the fundamental output voltage V_1 in terms of the switching angles is given by

$$V_1 = \frac{4V_{dc}}{\pi}\left(\cos(\alpha_1) + \cos(\alpha_2) + ... + \cos(\alpha_S)\right) \quad (6)$$

The relation between the fundamental voltage and the maximum obtainable fundamental voltage V_{1max} is given by modulation index. The modulation index, m_i, is defined as the ratio of the fundamental output voltage V_1 to the maximum obtainable fundamental voltage V_{1max}.

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The maximum fundamental voltage is obtained when all the switching angles are zero [4]. From equation (6),

$$V_{1\max} = \frac{4SV_{dc}}{\pi} \quad (7)$$

$$\therefore m_i = \frac{V_1}{V_{1\max}} = \frac{\pi V_1}{4SV_{dc}}$$ Consequently,

$$V_1 = m_i\left(\frac{4SV_{dc}}{\pi}\right) \quad \text{for } 0 < m_i \le 1 \quad (8)$$

To develop an 11-level cascaded multilevel inverter, five SDCSs are required. The modulation index and switching angles that result in the synthesis of AC waveform with the least Total Harmonic Distortion (THD) can be found by solving the following transcendental nonlinear equations known as SHE equations that characterize the selected harmonics:

$$\frac{4V_{dc}}{\pi}(\cos(\alpha_1)+\cos(\alpha_2)+\ldots+\cos(\alpha_5)) = V_1$$

$$\cos(5\alpha_1)+\cos(5\alpha_2)+\ldots+\cos(5\alpha_5) = V_5$$

$$\cos(7\alpha_1)+\cos(7\alpha_2)+\ldots+\cos(7\alpha_5) = V_7$$

$$\cos(11\alpha_1)+\cos(11\alpha_2)+\ldots+\cos(11\alpha_5) = V_{11}$$

$$\cos(13\alpha_1)+\cos(13\alpha_2)+\ldots+\cos(13\alpha_5) = V_{13} \quad (9)$$

In equation (10), V_5, V_7, V_{11}, and V_{13} are set to zero to in order to eliminate 5^{th}, 7^{th}, 11^{th} and 13^{th} harmonics respectively. The correct solution must satisfy the condition

$$0 \le \alpha_1 < \alpha_2 < \ldots < \alpha_5 \le \pi/2 \quad (10)$$

Equation (8) in equation (10) yields:

$$\cos(\alpha_1)+\cos(\alpha_2)+\ldots+\cos(\alpha_5) = 5m_i$$
$$\cos(5\alpha_1)+\cos(5\alpha_2)+\ldots+\cos(5\alpha_5) = 0$$
$$\cos(7\alpha_1)+\cos(7\alpha_2)+\ldots+\cos(7\alpha_5) = 0$$
$$\cos(11\alpha_1)+\cos(11\alpha_2)+\ldots+\cos(11\alpha_5) = 0$$
$$\cos(13\alpha_1)+\cos(13\alpha_2)+\ldots+\cos(13\alpha_5) = 0 \quad (11)$$

Generally equation (12) can be written as

$$F(\alpha) = B(m_i) \quad (12)$$

The Total Harmonic Distortion (THD) is computed as shown in equation (13):

$$THD = \sqrt{\sum_{i=5,7,11,13,\ldots}^{49}\left(\frac{V_i}{V_1}\right)^2} \quad (13)$$

III. PARTICLE SWARM OPTIMIZATION

Particle swarm optimization is a swarm intelligence based algorithm that was inspired by the social behavior in a flock of birds or a school of fish. The basic flowchart of PSO is shown in Fig.3

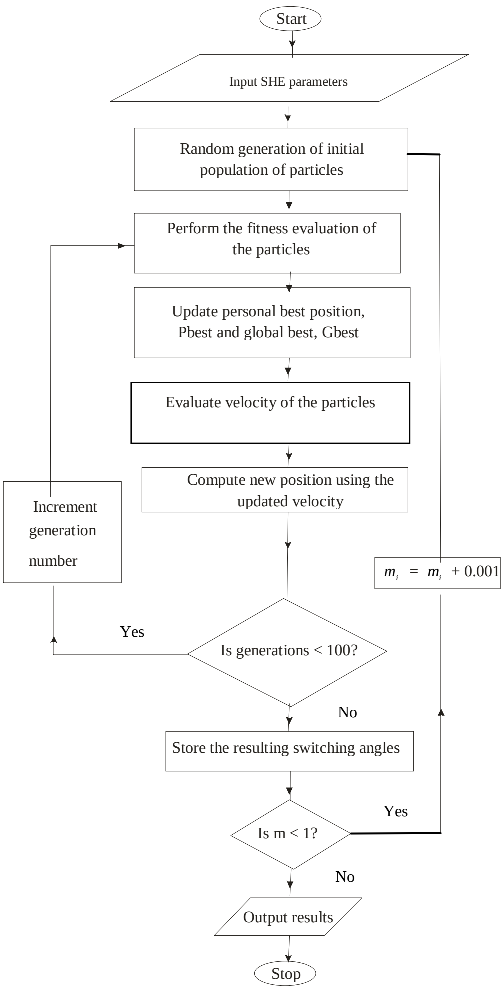

Fig. 2 Basic flowchart of PSO for SHE problem

In PSO, an initial population of potential solutions to the optimization problem called particles is randomly generated. Each particle in a swarm searches for the best position in the search space, while the social behavior that is modeled in PSO guide the swarm to the optimal region. Each particle in the search space is assigned a randomized position and velocity. During successive iteration, the current position of each particle in the swarm is evaluated with an objective function, and each particle keeps track of its coordinates in the problem space, which are associated with the best solution (fitness) that it has achieved so far. Based on the fitness evaluation of all the particles, the best position so far of the i^{th} particle in a d-dimensional space is called personal best (*Pbest*), and is denoted by $P_i = [p_{i1}, p_{i2}, \ldots, p_{id}]$, while the overall best position obtained so far by any particle in the swarm is called global best (*Gbest*), and is denoted by $P_g = [p_{g1}, p_{g2}, \ldots, p_{gd}]$. This implies that each particle has a memory which enables it to update its current position and velocity according to the distance between its current position and *Pbest*, as well as the distance between its current position and *Gbest.* If the velocity and position vectors of the i^{th} particle at iteration k are represented as

$V_i = [v_{i1}, v_{i2}, \ldots, v_{id}]$

and $X_i = [x_{i1}, x_{i2}, \ldots, x_{id}]$,

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respectively, then the velocity and position of the particle in the next iteration are determined as follows:

$$v_i(k+1) = wv_i(k) + c_1 r_1[p_i(k) - x_i(k)] + c_2 r_2[p_g(k) - x_i(k)] \quad (15)$$

$$x_i(k+1) = x_i(k) + v_i(k+1) \quad (16)$$

where w is the inertia weight parameter that provides the balance between global exploration and local exploitation capabilities of the particle, $v_i(k)$ is the velocity of the particle at iteration k; $x_i(k)$ is the position the particle at iteration k; c_1 and c_2 are constants known as cognitive and social coefficients, respectively; r_1 and r_2 are random values uniformly distributed within [0, 1] [18].

IV. IMPLEMENTATION

The solution set at each step is evaluated with the fitness function. The objective here is to determine the switching angles such that the selected low order harmonics are either eliminated or minimized to an acceptable level while the fundamental voltage is obtained at a desired value. For each solution set, the fitness function is calculated as follows [12]:

$$f = \min_{\alpha_i}\left[\left(100\frac{V_1^* - V_1}{V_1^*}\right)^4 + \sum_{s=2}^{5}\frac{1}{h_s}\left(50\frac{V_{hs}}{V_1}\right)^2\right] \quad i = 1, 2 \ldots 5 \quad (17)$$

subject to $0 \le \alpha_1 < \alpha_2 \ldots < \alpha_5 \le \pi/2$

Where V_1^* is the desired fundamental output voltage, S is the number of switching angles, h_s is the order of the s^{th} viable harmonic at the output of a three phase multilevel converter. For example, $h_2 = 5$, $h_3 = 7$. It should be noted that different weight are assigned to different harmonics. The fitness function assigns Eq. (17) assigns higher importance to the low order harmonics, which are more harmful and difficulty to remove with filter.

Using MATLAB software, the proposed PSO algorithm was implemented to eliminate 5th, 7th, 11th, and 13th harmonics, which are the low order harmonics an 11-level inverter. The choice of PSO algorithm's parameters is a tradeoff between accuracy and convergent speed. Due to the inexact nature of PSO algorithm, its parameters were chosen on trial and error basis. Based on the performance evaluation after several trials, the population size in this work is 40, the number of iterations is 100, and the values of both cognitive and social coefficients are chosen to be 1.5. Sometimes, PSO converges to a solution before 100 iterations are completed. In order to improve the convergent speed, iterations are stopped if the result remains unchanged for 50 iterations. The solutions were computed by incrementing the modulation index, m_i in steps of 0.001 from 0 to 1. A personal computer (2.66 GHz Intel Core i7 processor with 4GB Random Access Memory) running MATLAB R2014b on OS X Yosemite version 10.10 was used to carry out the computations. The algorithm was run several times, and the solution with the least fitness function was chosen.

The observed analytical results were validated with modelling and simulation of an 11-level single-phase Cascaded H-Bridge inverter using SimPower System block set in MATLAB-SIMULINK. In each of the five H-Bridges in the 11-level single-phase Cascaded H-Bridge inverter, 12V dc source is the SDCS, and the switching device used is Insulated Gate Bipolar Transistor (IGBT). Simulations were performed at the fundamental frequency of 50 Hz using the solution set found at the modulation index m_i of 0.92: $\alpha_1 = 3.76^o$, $\alpha_2 = 8.38^o$, $\alpha_3 = 19.43^o$, $\alpha_4 = 25.37^o$, and $\alpha_5 = 40.40^o$. Fourier Transform analysis of the simulated phase voltage waveforms was done using the FFT block to show the harmonic spectrum of the synthesized AC voltage.

V. RESULTS

For an 11-level inverter, variation of the fitness function with modulation index over the range of 0.1 to 1.0 is shown in Fig. 4. Solution sets with fitness value greater than 10^{-2} are rejected. When the fitness function at a modulation index is 10^{-2} or less, the corresponding switching angles are considered as a solution set.

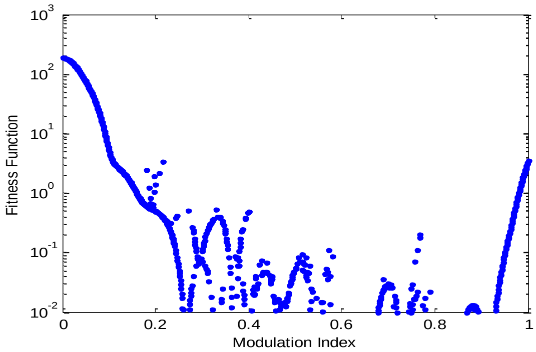

Fig. 4. Fitness function versus modulation index for 11-level inverter

Shown in Fig. 5 is the plot of switching angles that minimize 5th, 7th, 11th, and 13th harmonics in an 11-level inverter. It can be seen from Fig. 5 that there are multiple solution sets at some modulation indices. In the case of multiple solution sets, the set with the least THD is chosen. As can be observed from the THD curves of the solution sets plotted in Fig. 6, values of the 49th order THD are higher at lower modulation indices while they are considerably reduced at the upper end of modulation index. The plot of 13th order THD shows how efficiently the selected harmonics are minimized.

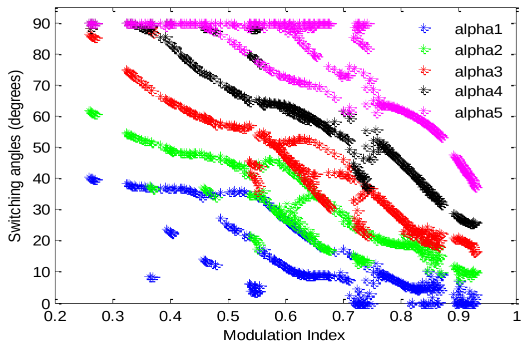

Fig. 5. Switching angles versus modulation index for 11-level inverter

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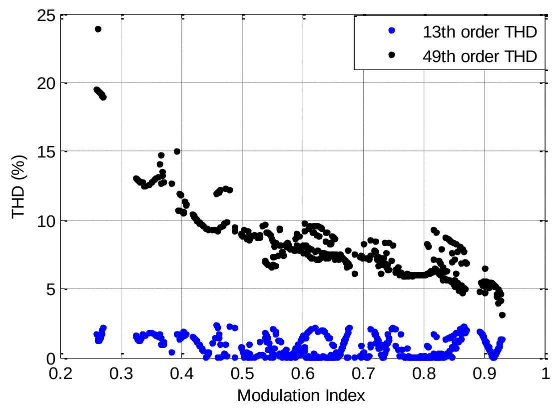

Fig. 6. THD versus modulation index for 11-level inverter

The analytically computed peak value of the fundamental output voltage given by eqn. (8) is

$$V_1 = m_i\left(\frac{4sV_{dc}}{\pi}\right) = 0.92\left(\frac{4\times 5\times 12}{\pi}\right) = 70.28V_{(peak)}$$

This value closely agrees with the simulation value of 70.22V shown in Fig. 7. The harmonic spectrum of the synthesized voltage waveform shown in Fig. 7 reveals the complete elimination of the $5^{th}, 7^{th}, 11^{th}$ *and* 13^{th} harmonics as their values tend towards zero.

The THD in line-to-line voltage as computed analytically with eqn. (13), and from simulation are 4.61% and 3.99% respectively. It should be noted that the simulation value of THD is shown in Fig. 7 as 16.55%. This value is for the phase voltage which includes triplen harmonic components while analytical value is for line voltage which excludes the triplen harmonics.

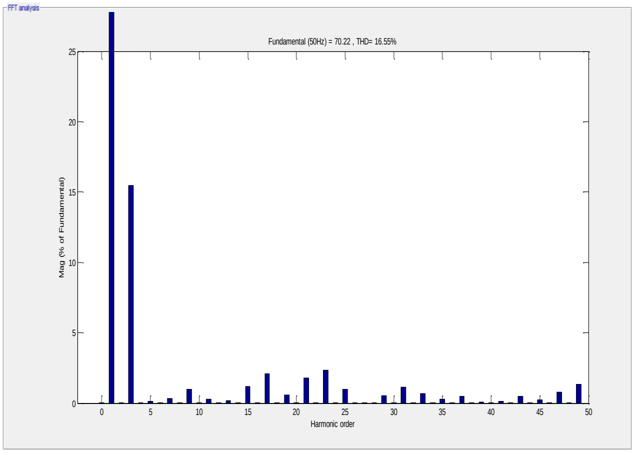

Fig.7. Harmonic spectrum for 11-level inverter at modulation index, $m_i = 0.92$

VI. CONCLUSION

PSO algorithm with random initial values has been successfully implemented for solving the transcendental nonlinear equations characterizing the selected harmonics in an 11-level inverter. The proposed method is derivative-free, accurate and globally convergent. Compared with the related works [4], [5] reported in literature, solution sets with well-attenuated low order harmonics have been found in the regions that are infeasible for analytical methods to find solution. Both analytical and simulation results are in close agreement. The absence of the selected harmonics in the synthesized output phase voltage validates the analytically computed results.

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