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Chromatic Values of Intuitionistic Fuzzy Directed Hypergraph Colorings

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Abstract*— A hypergraph is a set V of vertices and a set E of non-empty subsets of V, called hyperedges. Unlike graphs, hypergraphs can perform higher-order interactions in social and communication networks. Directed hypergraphs are much like directed graphs. Colors are used to distinguish the classes. Coloring a hypergraph H must assign atleast two different colors to the vertices of every hyperedge. That is, no edge is monochromatic. In this paper, upper and lower truncation, core aggregate of intuitionistic fuzzy directed hypergraph (IFDHG), conservative K-coloring of IFDHG, chromatic values of intuitionistic fuzzy colorings, elementary center of intuitionistic fuzzy coloring, f-chromatic value of intuitionistic fuzzy coloring, intersecting IFDHG, K-intersecting IFDHG, strongly intersecting IFDHG were studied. Also it has been proved that IFDHG H is strongly intersecting if and only if it is K-intersecting.

Index Terms*—Core aggregate of IFDHG, intuitionistic fuzzy colorings (IFC), elementary center, f -chromatic value of IFC, intersecting IFDHG, K-intersecting, strongly intersecting IFDHG.

I. INTRODUCTION

Fuzzy sets (FSs) introduced by L.A.Zadeh in 1965 [12] are generalization of crisp sets. K.T.Atanassov introduced the concept of intuitionistic fuzzy sets (IFSs) in 1999 [1] as an extension of FSs. These sets include not only the membership of the set but also the non-membership of the set along with degree of uncertainty. In order to expand the application base, the notion of graph was generalized to that of a hypergraph. In 1976, Berge [2] introduced the concepts of graph and hypergraph. This paper contains a few extensions of concepts in fuzzy hypergraphs by John N. Mordeson and Premchand S. Nair [3]. The paper has been organized as follows: Section 2 deals with the definitions of fuzzy hypergraph, intuitionistic fuzzy hypergraph, IFDHG and the notations used in this paper. In section 3 and 4, a study is made on core aggregate of IFDHG, conservative K-coloring of intuitionistic fuzzy directed hypergraph, chromatic values of intuitionistic fuzzy colorings, elementary center of intuitionistic fuzzy coloring, f -chromatic value of intuitionistic fuzzy coloring, intersecting IFDHG, K-intersecting IFDHG, strongly intersecting IFDHG. Some properties of the newly proposed hypergraph concepts are also discussed. Section 5 concludes the paper.

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II. NOTATIONS AND PRELIMINARIES

The notations used in this work are listed below:

$H = (V, E)$	- IFDHG with vertex set V and edge set E
$\langle \mu_i, \nu_i \rangle$	- degrees of membership and non-membership of the vertex
$\langle \mu_{ij}, \nu_{ij} \rangle$	- degrees of membership and non-membership of the edges
$\langle \mu_{ij}(v_i), \nu_{ij}(v_i) \rangle$	- degrees of membership and non-membership of the edges containing v_i
$h(H)$	- Height of a hypergraph H
$F(H)$	- Fundamental sequence of H
$C(H)$	- Core set of H
$H^{(r_i, s_i)}$	- (r_i, s_i) - level of H
$IF_p(v)$	- IF power set of V.

In this section, definitions of intuitionistic fuzzy set, intuitionistic fuzzy graph, IFDHG are dealt with.

Definition 2.1. [1] Let a set E be fixed. An *intuitionistic fuzzy set (IFS)* V in E is an object of the form $V = \{\langle v_i, \mu_i(v_i), \nu_i(v_i) \rangle / v_i \in E\}$, where the function $\mu_i : E \to [0, 1]$ and $\nu_i : E \to [0, 1]$ determine the degree of membership and the degree of non-membership of the element $v_i \in E$, respectively and for every $v_i \in E$, $0 \le \mu_i(v_i) + \nu_i(v_i) \le 1$.

Definition 2.2. [1] The five Cartesian products of two IFSs V_1, V_2 of V over E is defined as

$$V_1 \times_1 V_2 = \{\langle (v_1, v_2), \mu_1.\mu_2, \nu_1.\nu_2 \rangle | v_1 \in V_1, v_2 \in V_2\},$$

$$V_1 \times_2 V_2 = \{\langle (v_1, v_2), \mu_1 + \mu_2 - \mu_1\mu_2, \nu_1.\nu_2 \rangle | v_1 \in V_1, v_2 \in V_2\},$$

$$V_1 \times_3 V_2 = \{\langle (v_1, v_2), \mu_1.\mu_2, \nu_1\nu_2 - \nu_1.\nu_2 \rangle | v_1 \in V_1, v_2 \in V_2\},$$

$$V_1 \times_4 V_2 = \{\langle (v_1, v_2), \min(\mu_1, \mu_2), max(\nu_1, \nu_2) \rangle | v_1 \in V_1, v_2 \in V_2\},$$

$$V_1 \times_5 V_2 = \langle (v_1, v_2), \max(\mu_1, \mu_2), min(\nu_1, \nu_2) \rangle | v_1 \in V_1, v_2 \in V_2\}$$

It must be noted that $v_i \times_s v_j$ is an IFS, where s = 1, 2, 3, 4, 5.

Definition 2.3. [1] Let E be fixed set and $V = \{\langle v_i, \mu_i(v_i), \nu_i(v_i) \rangle / v_i \in E\}$, be an IFS. Six types of Cartesian products of n subsets[1] $V_1, V_2 \ldots . V_n$ of V over E are defined as

[1]subsets -crisp sense

$$V_1 \times_1 V_2 \times_1 \ldots \times_1 V_n = \{\langle (v_1, v_2 \ldots v_n), \textstyle\prod_{i=1}^{n} \mu_i, \prod_{i=1}^{n} \nu_i \rangle | v_1 \in V_1, v_2 \in V_2 \ldots v_n \in V_n\},$$

$V_{i_1} \times_2 V_{i_2} \times_2 \dots \times_2 V_{i_n}$
$= \left\{ \langle (v_1, v_2 \dots v_n), \sum_{i=1}^{n} \mu_i - \sum_{i=1}^{n} \mu_i \mu_j - \sum_{i \neq j \neq k} \mu_i \mu_j \mu_k - \dots + (-1)^{n-2} \sum_{i \neq j \neq k \neq \dots \neq n} \mu_i \mu_j \mu_k \dots \mu_n + (-1)^{n-1} \prod_{i=1}^{n} \mu_i, \prod_{i=1}^{n} v_i \rangle \middle| v_1 \in V_1, v_2 \in V_2, \dots v_n \in V_n \right\}$

$V_{i_1} \times_3 V_{i_2} \times_3 \dots \times_3 V_{i_n}$
$= \left\{ \langle (v_1, v_2 \dots v_n), \prod_{i=1}^{n} \mu_i, \sum_{i=1}^{n} v_i - \sum_{i \neq j}^{n} v_i v_j + \sum_{i \neq j \neq k}^{n} v_i v_j v_k - \dots + (-1)^{n-2} \sum_{i \neq j \neq k \neq \dots \neq n} v_i v_j v_k \dots v_n + (-1)^{n-1} \prod_{i=1}^{n} v_i \rangle \middle| v_1 \in V_1, v_2 \in V_2, \dots v_n \in V_n \right\}$

$V_{i_1} \times_4 V_{i_2} \times_4 \dots \times_4 V_{i_n}$
$= \{ \langle (v_1, v_2 \dots v_n), \min(\mu_1, \mu_2, \dots \mu_n), max(v_1, v_2, \dots v_n) \rangle | v_1 \in V_1, v_2 \in V_2, \dots v_n \in V_n \}$

$V_{i_1} \times_5 V_{i_2} \times_5 \dots \times_5 V_{i_n}$
$= \{ \langle (v_1, v_2 \dots v_n), \max(\mu_1, \mu_2, \dots \mu_n), min(v_1, v_2, \dots v_n) \rangle | v_1 \in V_1, v_2 \in V_2, \dots v_n \in V_n \}$

$V_{i_1} \times_6 V_{i_2} \times_6 \dots \times_6 V_{i_n}$
$= \{ \langle (v_1, v_2 \dots v_n), \frac{\sum_{i=1}^{n} \mu_i}{2}, \frac{\sum_{i=1}^{n} v_i}{2} \rangle | v_1 \in V_1, v_2 \in V_2, \dots v_n \in V_n \},$

It must be noted that $v_i \times_s v_j$ is an IFS, where s = 1, 2, 3, 4, 5,6.

Definition 2.4. [4] An *intuitionistic fuzzy graph (IFG)* is of the form $G = (V, E)$ where (i)$V = \{v_1, v_2, \dots v_n\}$ such that $\mu : E \to [0,1]$ and $\nu : E \to [0,1]$ denote the degrees of membership and non-membership of the vertex $v_i \in V$ respectively and

$0 \leq \mu_i(v_i) + \nu_i(v_i) \leq 1$ ----------(1)

for every $v_i \in V, i = 1,2,3 \dots n$. (ii) $E \subseteq V \times V$ where $\mu_{ij} : V \times V \to [0,1]$ and $\nu_{ij} : V \times V \to [0,1]$ are such that

$\mu_{ij} \leq \mu_i \emptyset \mu_j$ ------------(2)

$\nu_{ij} \leq \nu_i \emptyset \nu_j$ ------------(3)

and $0 \leq \mu_{ij} + \nu_{ij} \leq 1$ ------------(4)

where μ_{ij} and ν_{ij} are the degrees of membership and non-membership of the edge (v_i, v_j);the values of $\mu_i \emptyset \mu_j$ and $\nu_i \emptyset \nu_j$ can be determined by one of the cartesian products $\times_s, s = 1,2,3, \dots 6$ for all *i* and *j* given in Definition 2.2.

Note:

Throughout this paper, it is assumed that the fifth Cartesian product in Definition 2.3

$V_{i_1} \times_5 V_{i_2} \times_5 \dots \times_5 V_{i_n}$
$= \{ \langle (v_1, v_2 \dots v_n), \max(\mu_1, \mu_2, \dots \mu_n), min(v_1, v_2, \dots v_n) \rangle | v_1 \in V_1, v_2 \in V_2, \dots v_n \in V_n \}$

is used to determine the degrees of membership μ_{ij} and non-membership ν_{ij} of the edge e_{ij}.

Definition 2.5. [5] An *intuitionistic fuzzy hypergraph* (IFHG) is an ordered pair H = (V, E) where

(i) $V = \{v_1, v_2, \dots, v_n\}$, is a finite set of intuitionistic fuzzy vertices,

(ii) $E = \{E_1, E_2, \dots, E_m\}$ is a family of crisp subsets of V

(iii) $E_j = \{(v_i, \mu_j(v_i), \nu_j(v_i)) : \mu_j(v_i), \nu_j(v_i) \geq 0$ and $\mu_j(v_i), \nu_j(v_i) \leq 1\}, j = 1,2, \dots, m,$

(iv) $E_j \neq \phi, j = 1,2,3, \dots m$.

Here, the hyperedges E_j are crisp sets of intuitionistic fuzzy vertices $\mu_j(v_i)$ and $\nu_j(v_i)$ denote the degrees of membership and non-membership of vertex v_i to edge E_j. Thus, the elements of the incidence matrix of IFHG are of the form $\left(v_{ij}, \mu_j(v_i), \nu_j(v_i)\right)$. The sets *(V,E)* are crisp sets.

Note:The support of an IFS V in E is denoted by $supp(E_j) = \{v_i / \mu_{ij}(v_i) > 0$ and $\nu_{ij}(v_i) > 0\}$.

Definition 2.6. [6] An intuitionistic fuzzy directed hypergraph (IFDHG) H is a pair *(V,E)*, where V is a non - empty set of vertices and E is a set of intuitionistic fuzzy hyperarcs; an intuitionistic fuzzy hyperarc $E_i \in E$ is defined as a pair $t(E_i), h(E_i))$, where $(E_i) \subset V$, with $t(Ei) \neq \phi$, is its tail, and $h(E_i) \in V - t(E_i)$ is its head. A vertex *s* is said to be a source vertex in *H* if $h(E_i) \neq s$, for every $E_i \in E$. A vertex *d* is said to be a destination vertex in H if $d \neq t(E_i)$, for every $E_i \in E$.

Definition 2.7.[11] Let H be an IFDHG, let $H^{r_i,s_i} = (V^{r_i,s_i}, E^{r_i,s_i})$ be the $\langle r_i, s_i \rangle$ -level IFDHG of *H*. The sequence of real numbers $\{r_1, r_2, \dots, r_n; s_1, s_2, \dots, s_n\}$, such that $0 \leq r_i \leq h_\mu(H)$ and $0 \leq s_i \leq h_\nu(H)$, satisfying the properties:

(i) If $r_1 < \alpha \leq 1$ and $0 \leq \beta < s_1$ then $E^{\alpha,\beta} = \varphi$,

(ii) If $r_i + 1 \leq \alpha \leq r_i$; $s_i \leq \beta \leq s_i + 1$ then

$$E^{\alpha,\beta} = E^{r_i,s_i}$$

(iii) $E^{r_i,s_i} \sqsubset E^{r_{i+1},s_{i+1}}$

is called the fundamental sequence of *H*, and is denoted by *F(H).* The core set of *H* is denoted by *C(H)* and is defined by $C(H) = \{H^{r_1,s_1}, H^{r_2,s_2}, \dots, H^{r_n,s_n}\}$. The corresponding set of $\langle r_i, s_i \rangle$- level hypergraphs $H^{r_1,s_1} \subset H^{r_2,s_2} \subset \dots \subset H^{r_n,s_n}$ is called the *H* induced fundamental sequence and is denoted by *I(H).* The $\langle r_n, s_n \rangle$- level is called the support level of *H* and the H^{r_n,s_n} is called the support of *H*.

Definition 2.8.[11] Let *H* be an IFDHG and $C(H) = \{H^{r_1,s_1}, H^{r_2,s_2}, \dots, H^{r_n,s_n}\}$. *H* is said to be ordered if *C(H)* is ordered. That is $H^{r_1,s_1} \subset H^{r_2,s_2} \subset \dots \subset H^{r_n,s_n}$. The IFDHG is said to be simply ordered if the sequence $\{H^{r_i,s_i} / i = 1,2,3 \dots, n\}$ is simply ordered, that is if it is ordered and if whenever $E \in H^{r_{i+1},s_{i+1}} - H^{r_i,s_i}$ then $E \nsubseteq H^{r_i,s_i}$.

Definition 2.9.[11] A minimal intuitionistic fuzzy transversal T for H is a transversal of H with the property that if $T_1 \subset T$, then T_1 is not an intuitionistic fuzzy transversal of *H*.

III. CHROMATIC VALUES OF INTUITIONISTIC FUZZY DIRECTED HYPERGRAPH COLORINGS

Let $H = (V,E)$ be an intuitionistic fuzzy directed hypergraph (IFDHG).

Definition 3.1. Let H be an IFDHG. The lower truncation H_l of H at $\langle r_l, s_l\rangle$ -level, $0 < r_l \leq h_\mu(H)$, $0 < s_l \leq h_\nu(H)$, where $r_l < \mu_i$, $s_l < \nu_i$, forall v_i, is an IFDHG, $H_l = < V_t, E_t, \mu_{t_l}(e_{ij}), \nu_{t_l}(e_{ij}) >$, where $V_t \subset V$ and $E_t \subset E$ denote the sets of vertices and edges of truncated IFDHG respectively and

$$\mu_{t_l}(e_{ij}) = \begin{cases} \mu_{ij} & \text{if } \mu_{ij} \geq r_l \\ 0 & \text{otherwise} \end{cases}$$
$$\nu_{t_l}(e_{ij}) = \begin{cases} \nu_{ij} & \text{if } \nu_{ij} \leq s_l \\ 0 & \text{otherwise} \end{cases}$$

are the membership and non-membership values of the edge e_{ij}.

The upper truncation H_u of H at $\langle r_u, s_u\rangle$-level, $0 < r_u \leq h_\mu(H)$, $0 < s_u \leq h_\nu(H)$, where $r_u < \mu_i$, $s_u < \nu_i$, forall v_i, is an IFDHG, $H_u = < V_t, E_t, \mu_{t_u}(e_{ij}), \nu_{t_u}(e_{ij}) >$, where $V_t \subset V$ and $E_t \subset E$ denote the sets of vertices and edges of truncated IFDHG respectively and

$$\mu_{t_u}(e_{ij}) = \begin{cases} \mu_{ij} & \text{if } \mu_{ij} \geq r_u \\ 0 & \text{otherwise} \end{cases}$$
$$\nu_{t_u}(e_{ij}) = \begin{cases} \nu_{ij} & \text{if } \nu_{ij} \leq s_u \\ 0 & \text{otherwise} \end{cases}$$

are the membership and non-membership values of the edge e_{ij}.

Note: μ_{t_l}, μ_{t_u} are degrees of membership values of lower and upper truncation, ν_{t_l}, ν_{t_u} are degrees of non-membership values of lower and upper truncation.

Example 1. Consider an IFDHG, H with the adjacency matrix as given below:

$$H = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \end{array} \begin{array}{ccc} E_1 & E_2 & E_3 \\ \langle 0.7,0.3\rangle & \langle 0,1\rangle & \langle 0.7,0.3\rangle \\ \langle 0.6,0.3\rangle & \langle 0.6,0.3\rangle & \langle 0,1\rangle \\ \langle 0,1\rangle & \langle 0.5,0.2\rangle & \langle 0,1\rangle \\ \langle 0,1\rangle & \langle 0,1\rangle & \langle 0.3,0.3\rangle \\ \langle 0.6,0.2\rangle & \langle 0.6,0.2\rangle & \langle 0.6,0.2\rangle \end{array}$$

The corresponding graph of IFDHG H is displayed in Figure 1.

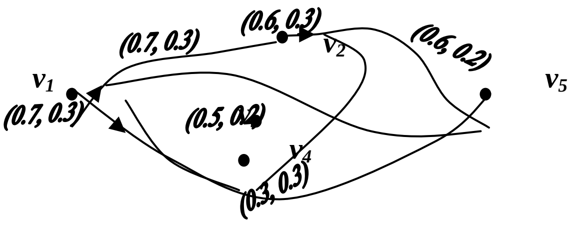

Figure. 1: Intuitionistic fuzzy directed hypergraph *H*

The adjacency matrix of lower truncation of $H^{(0.6,0.3)}$ is given by

$$H = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \end{array} \begin{array}{ccc} E_1 & E_2 & E_3 \\ \langle 0.7,0.3\rangle & \langle 0,1\rangle & \langle 0.7,0.3\rangle \\ \langle 0.6,0.3\rangle & \langle 0.6,0.3\rangle & \langle 0,1\rangle \\ \langle 0,1\rangle & \langle 0,1\rangle & \langle 0,1\rangle \\ \langle 0,1\rangle & \langle 0,1\rangle & \langle 0,1\rangle \\ \langle 0,1\rangle & \langle 0,1\rangle & \langle 0,1\rangle \end{array}$$

The adjacency matrix of upper truncation of $H^{(0.6,0.3)}$ is given by

$$H = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \end{array} \begin{array}{ccc} E_1 & E_2 & E_3 \\ \langle 0,1\rangle & \langle 0,1\rangle & \langle 0,1\rangle \\ \langle 0,1\rangle & \langle 0,1\rangle & \langle 0,1\rangle \\ \langle 0,1\rangle & \langle 0.5,0.2\rangle & \langle 0,1\rangle \\ \langle 0,1\rangle & \langle 0,1\rangle & \langle 0.3,0.3\rangle \\ \langle 0,1\rangle & \langle 0,1\rangle & \langle 0,1\rangle \end{array}$$

The graphs of lower and upper truncations are given in Figure 2:

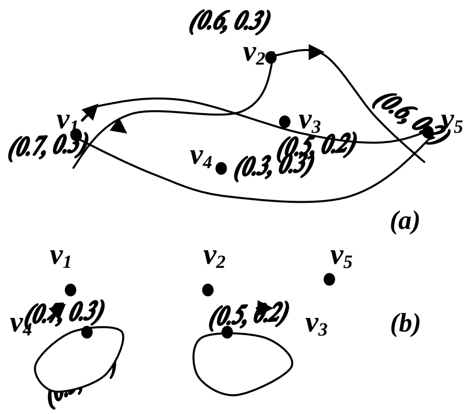

Figure. 2: (a) Lower Truncation of $H^{(0.6,0.3)}$ and (b) Upper Truncation of $H^{(0.6,0.3)}$

Note:
1. $H_l \cup H_u \subseteq H$.
2. $H_l \cap H_u = \varphi$.

Definition 3.2. Let H be an IFDHG with core set

$$C(H) = \{H^{r_i,s_i} = (V^{r_i,s_i}, E^{r_i,s_i})/i = 1,2,\ldots n\}$$

where $E(H^{r_i,s_i}) = E_i$ is the crisp edge set of the core hypergraph H^{r_i,s_i}. Let $E(H)$ denote the crisp edge set of H defined by

$$E(H) = \cup\{E_i/E_i = E(H^{r_i,s_i}); H^{r_i,s_i} \in C(H)\}.$$

$E(H)$, a crisp hypergraph on V, is called *core aggregate hypergraph* of H and is denoted by $\mathcal{H}(H) = (V, E(H))$.

Theorem 3.1. For every intuitionistic fuzzy hypergraph H, a p-coloring of $\mathcal{H}(H)$ is a K-coloring of H and conversely.

Definition 3.3. Let H be an IFDHG. Then, every K-colorings of H which is a conservative p-coloring of $\mathcal{H}(H)$, is called a *conservative K-colorings of H.*

Definition 3.4. Let $H^{r_j,s_j} = (V^{r_j,s_j}, E^{r_j,s_j})$ be a (crisp) core directed hypergraph of an IFDHG H where $V^{r_i,s_i} = V \backslash V^{r_j,s_j} \neq \phi$. Suppose L is a K-colorings of upper truncated IFDHG, H^{r_j,s_j} which is obtained by extending a p-coloring, L_j of H^{r_j,s_j}. If L is weekly (or strongly) conservative p-coloring extension of L_j to the (crisp) core aggregate hypergraph $H(H^{r_j,s_j})$ of H^{r_j,s_j} with respect to V^{r_i,s_i}, then L is called a weekly (or strongly) conservative K-coloring extension of L_j with respect to V^{r_i,s_i}.

Definition 3.5. Let H be an IFDHG and suppose $\Lambda = \{\delta_i \in IF_P(v)/i = 1,2,3 \ldots p\}$ is a finite subset of $IF_P(v)$. Then Λ is called *intuitionistic fuzzy coloring of H* if the following properties are satisfied:

1) $h(\Lambda) = \langle max(\mu_{ij}(v_i)), \min(\nu_{ij}(v_i))\rangle$, for all $v_i \in V$
2) $\delta_i \cap \delta_j = \phi$ if $i \neq j$
3) Λ^{r_i,s_i} is a coloring of H^{r_i,s_i} for $0 \leq r_i \leq h_\mu(H)$ and $0 \leq s_i \leq h_\nu(H)$.

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Theorem 3.2. For every intuitionistic fuzzy hypergraph H, a p-coloring of $\mathcal{H}(H)$ is a K-coloring of H and vice-versa.

Note: Λ is sequentially elementary with respect to *F(H)*. There is one-to-one correspondence between the K-coloring of H and the intuitionistic fuzzy coloring of H, if the color set is empty.

Let Λ is an IFC of $H = (V, E)$. Then by Definition 3.1, $\langle r_n, s_n\rangle$- cut, Λ^{r_n,s_n} of Λ, where $\langle r_n, s_n\rangle$ - is the smallest value in *F(H)*, is p-coloring of the core aggregate hypergraph $\mathcal{H}(H)$ of H which implies Λ^{r_n,s_n} is a $K-$ coloring of H by Theorem 3.1.

Conversely, suppose $A = \{A_1, A_2, \ldots A_k\}$ is a K-coloring of $\mathcal{H}(H)$. Then A is a crisp coloring of the core aggregate hypergraph $\mathcal{H}(H)$ of H, $\cup_{i=1}^{k} A_k = V$ and $A_i \cap A_j = \phi$ if $i \neq j$. Now A_i, associate an intuitionistic fuzzy subset $\delta_i \in IF_P(v)$, with support A_i, defined by

$$\delta_i(v_i) = \begin{cases} \langle \vee (\mu_{ij}(v_i), \wedge \left(\nu_{ij}(v_i)\right)\rangle & if\ v_i \in A_i \\ \langle 0,1\rangle & \text{otherwise} \end{cases}$$

for all $\mu_{ij}, \nu_{ij} \in E$. Hence $\Lambda = \{\delta_1, \delta_2, \ldots, \delta_k\}$ is an IFC of H.

Definition 3.6. Let $\delta_i \in IF_P(v)$. Then the intuitionistic fuzzy subset $\delta_{i(c)}$ of V for all $v_i \in V$ is defined by

$$\delta_{i(c)} = \begin{cases} h(\delta) \text{ if } \delta_i(v_i) = \langle \vee (\mu_{ij}(v_i), \wedge \left(\nu_{ij}(v_i)\right)\rangle \\ \langle 0, 1\rangle \quad otherwise \end{cases}$$

for all $\mu_{ij}, \nu_{ij} \in E$. $\delta_{i(c)}$ is called *the elementary center*.

Definition 3.7 Let $\Lambda = \{\delta_i \in IF_P(v)/i = 1, 2, \ldots. p\}$. Then $\Lambda_{i(c)}$ is called *elementary center of* Λ, is defined by $\Lambda_{i(c)} = \{\delta_{1(c)}, \delta_{2(c)}, \ldots. \delta_{p(c)}$ where $\delta_{i(c)}$ is the elementary center of δ_i.

Definition 3.8. Let $\Lambda_{(c)}$ be the elementary center of IFC Λ of H with fundamental sequence $F(\Lambda_{(c)}) = \{u_1^{\Lambda}, u_2^{\Lambda}, \ldots, u_m^{\Lambda}\}$, where $u_1^{\Lambda} > u_2^{\Lambda} > \cdots > u_m^{\Lambda}$ and let t be a monotonic increasing function on the interval [0, 1] such that $t(0) = 0$ and $t(1) = 1$. Such t is called *scaling function*.

Definition 3.9. Let H be an IFDHG and let t denote a scaling function. Then $= min\{$⍰$r_t(\Lambda)/\Lambda$ is an IFC of $H\}$ and $\hat{\chi}_t(H) = min\{$⍰$\hat{r}_t(\Lambda)/\Lambda$ is an IFC of $H\}$ are called ⍰r_t*-chromatic number* and ⍰$\hat{r}_t$*-chromatic number* of H respectively.

Note: If t is the identity mapping on [0, 1], then ⍰r_t or ⍰$\hat{r}_t$ are called linear chromatic numbers of H.

Theorem 3.3. Let H be an IFDHG then for every H and for every scaling function $t : [0, 1] \to [0, 1]$, $\chi_t(H) \leq \chi(H)$, ⍰$r_t(H) \leq$ ⍰$\hat{r}_t(H)$ and χ(H)= min{| Λ |/ Λ is an IFC of H} = min{|L|/L is a K-coloring of H} where | Λ | is the number of edges in Λ and |L| is the number of colors in L.

Example 2. Consider an IFDHG, H with $V = \{v_1\ v_2, v_3, \ldots, v_9\}$ and $E = \{E_1\ E_2, E_3, \ldots, E_{15}\}$:

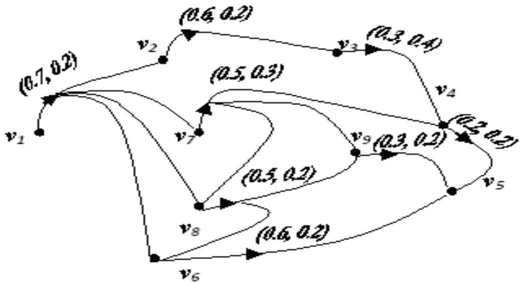

Figure. 3: Chromatic Numbers of H

Here $V = \{v_1\ v_2, v_3, \ldots, v_9\}$, $E = \{E_1\ E_2, E_3, \ldots, E_{15}\}$ and $C(H) = \{H^{r_1,s_1}, H^{r_2,s_2}\}$ where $H^{r_1,s_1} = (\{v_1\ v_2, v_3, \ldots, v_6\}, \{E_1\ E_2, E_3, \ldots, E_6\})$ and $H^{r_2,s_2} = (V, E)$.

Since H is elementary, it is ordered.

Thus every primitive coloring of H is an K-coloring of H. Therefore $\chi(H) = 3$, since H^{r_2,s_2} has the following primitive coloring:

$A_1 = \{Blue(B), Green(G), Yellow(Y)\}$ where $B = \{v_1, v_4, v_9\}, G = \{v_2, v_6, v_8\}$ and $Y = \{v_3, v_5, v_7\}$.

Suppose t is the identity map. Assume $\chi_t(H) =$ ⍰$r_t(\Lambda)$. It is interesting to compare ⍰$r_t(\Lambda_1)$. with ⍰$r_t(\Lambda_2)$, where Λ_1 and Λ_2 are the IFC of H.

Let $A_2 = \{Blue(B), Green(G), Y\,ellow(Y),$ $Red(R), White(W)\}$, where $B = \{v_1, v_3, v_5\}$, $G = \{v_7\}, R = \{v_8\}, W = \{v_9\}$ and $Y = \{v_2, v_4, v_6\}$. The restriction, A_2' of A_2 to H^{r_1,s_1} is $\chi(H) = 2$, $A_2' = \{B, Y\}$.

Definition 4.1. An IFDHG H is said to be an *intersecting intuitionistic fuzzy directed hypergraph*, if for each pair of intuitionistic fuzzy hyperedge $\{E_i, E_j\} \subseteq E$, $E_i \cap E_j \neq \phi$.

Definition 4.2. Let H be an IFDHG and

$$C(H) = \{H^{r_i,s_i} = (V^{r_i,s_i}, E^{r_i,s_i})/i = 1, 2, \ldots n\}$$

, if H^{r_i,s_i} is an intersecting IFDHG for each $i = 1, 2, \ldots, n$ then H is *K-intersecting IFDHG*.

Definition 4.3. An IFDHG H is said to be *strongly intersecting*, if for any two edges E_i and E_j contain a common spike of height, $h = h(E_i) \wedge h(E_j)$.

Theorem 4.1. Let H be an IFDHG and suppose $C(H) = \{H^{r_i,s_i} = (V^{r_i,s_i}, E^{r_i,s_i})/i = 1, 2, \ldots n\}$. Then H is intersecting if and only if $H^{r_n,s_n} = (V^{r_n,s_n}, E^{r_n,s_n})$ is intersecting.

Proof:

H is intersecting $\Leftrightarrow$ $supp(H) = \{supp(E_j)/E_j \in E\}$ is intersecting, from Definition 3.9. Similarly, each pair of intuitionistic fuzzy hyperedges,

$$\{E_1\ E_2, E_3, \ldots, E_6\} \subseteq E$$

$H^{r_1,s_1}, H^{r_2,s_2}, \ldots H^{r_n,s_n}$ are intersecting.

Conversely, let $H^{r_n,s_n} = (V^{r_n,s_n}, E^{r_n,s_n})$ is intersecting. Since, $supp(H) = \{supp(E_j)/E_j \in E\}$ is intersecting, H is also intersecting.

Theorem 4.2. Let H be an ordered intuitionistic fuzzy directed hypergraph and let $C(H) = \{H^{r_i,s_i} = (V^{r_i,s_i}, E^{r_i,s_i})/i = 1,2,\ldots n\}$, then H is intersecting if and only if H is K-intersecting.

Proof:

The proof is direct from Definition 3.10 and Theorem 3.3.

Theorem 4.3. Suppose H is an ordered intersecting IFDHG, then each intuitionistic fuzzy hyperedge T of H contains a member of $Tr(H_{h(T)})$, where $H_{h(T)}$ is the upper truncation of H at level *h(T)*. In particular T is an intuitionistic fuzzy transversal of $H_{h(T)}$.

Proof:

Let $C(H) = \{H^{r_i,s_i} = (V^{r_i,s_i}, E^{r_i,s_i})/i = 1,2,\ldots n\}$, and suppose $T_j \in E$. Assume that, $\langle r_1, s_1\rangle = h(T)$, since H is ordered and $T^{r_i,s_i} \neq \phi$. Since H is intersecting $\Rightarrow H^{r_n,s_n}$ is also intersecting. Therefore, T^{r_1,s_1} is an intuitionistic fuzzy transversal of H^{r_n,s_n}. Let T_1 be a minimal intuitionistic fuzzy transversal of H^{r_n,s_n} contained in T^{r_1,s_1}. Since H is ordered, then there is a nested sequence of sets

$$T_n \supseteq \ldots . T_i \supseteq \ldots . \supseteq T_1$$

such that, T_i is a minimal intuitionistic fuzzy transversal of H^{r_i,s_i} for every $\langle r_1, s_1\rangle \in F(H)$. Let θ_i be the elementary intuitionistic fuzzy subset with support T' and height $\langle r_i, s_i\rangle$, for $i = 1,2,\ldots,n$. Then, $\bigcup_{i=1}^{n} \theta_i \in Tr(H)$ and $T' \subseteq T$. Therefore, each intuitionistic fuzzy hyperedge T of H contains a member of $Tr(H_{h(T)})$.

Theorem 4.4. If H is a simple, intersecting IFDHG such that $\chi(H) > 2$, then $E = \{T' | T' \in min(Tr(H))\}$.

Corollary 4.5. If Theorem 3.6 holds good for $\chi(H) > 2$, then H has no loops.

Theorem 4.6 Let H be an ordered, intersecting IFDHG with $C(H) = \{H^{r_i,s_i} = (V^{r_i,s_i}, E^{r_i,s_i})/i = 1,2,\ldots n\}$. Suppose that $\chi(H^{r_i,s_i}) > 2$ and H^{r_n,s_n} is simple. Then for each $\langle r_i, s_i\rangle \in F(H)$,

$$Tr(H^{r_i,s_i}) = \{\theta(T, \langle r_i, s_i\rangle)/T \in H^{r_i,s_i}\}$$

where $\theta(T, \langle r_i, s_i\rangle)$ is an elementary intuitionistic fuzzy subset with support E and height $< r_i, s_i >$.

Proof:

By hypothesis, it follows that H^{r_i,s_i} is simple, intersecting and $\chi(H^{r_i,s_i}) > 2$ for each $H^{r_i,s_i} \in C(H)$.

By theorem 3.6, T is the set of all minimal transversals of H. Thus the set of $H^{r_i,s_i} = Tr(H^{r_i,s_i})$, for every $\langle r_i, s_i\rangle \in F(H)$. Hence the desired result.

Theorem 4.7. Let H be an IFDHG. Then H is strongly intersecting if and only if H is K-intersecting.

Proof:

Necessary Part: Suppose that H is strongly intersecting, let E_i and E_j be edges of $H^{r_i,s_i} \in C(H)$. Then there exists two edges E_1 and E_2 of H such that, $E_1^{r_i,s_i} = \mathrm{E}_1$ and $E_2^{r_i,s_i} = \mathrm{E}_2$.

Since H is strongly intersecting, both E_1 and E_2 contain a common spike θ_{v_i}, where $0 \leq r_i \leq h_\mu(\theta_{v_i})$ and $0 \leq s_i \leq h_\nu(\theta_{v_i})$. Thus, $(\theta_{v_i}) = \{v_i\} \subseteq E_i \cap E_j$. Hence H^{r_i,s_i} is intersecting and H is K - intersecting.

Sufficient Part: Suppose that H is K-intersecting, let F_i and F_j be hyperedges of H and let $\langle r_i, s_i\rangle = \mathrm{h}(F_i) \wedge \mathrm{h}(F_j)$ and let $E_i = F_i^{r_i,s_i}$, $E_j = F_j^{r_i,s_i}$, then both $E_i, E_j \in H^{r_i,s_i} = H^{r_j,s_j}$, where $r_{j+1} < r_i \leq r_j, s_{j+1} < s_i \leq s_j$. Let $\langle r_{n+1}, s_{n+1}\rangle = \langle 0,1\rangle$, since H^{r_i,s_i} is intersecting, there exists a vertex $v_i \in E_i \cap E_j$. There is a spike θ_{v_i} with support $\{v_i\}$ and height $\langle r_i, s_i\rangle$ which is contained in both F_i and F_j. Hence H is strongly intersecting.

Theorem 4.8. If H^s is intersecting, then H is strongly intersecting.

Proof:

Let $C(H) = \{H^{r_i,s_i} = (V^{r_i,s_i}, E^{r_i,s_i})/i = 1,2,\ldots n\}$ be the set of core intuitionistic fuzzy hypergraphs of H and consider the core's aggregate intuitionistic fuzzy hypergraph,

$\mathcal{H}(H) = (V, E(H))$, where $E(H) = \cup \{E_i | i = 1,2,\ldots,n\}$.

In addition, let $(H^S)^{r_m^S,s_m^S} = (V_m^S, E_m^S)$, represent the core hypergraph of H^S associated with the smallest member $\langle r_m^S, s_m^S\rangle$ of *F(H)*.

From the construction of H^S it follows that every edge belonging to *E(H)* contains an edge of E_m^S.

Hence H^S is intersecting $\Rightarrow H$ is strongly intersecting.

If H^S is intersecting, then by Theorem 3.6, $(H^S)^{r_m^S,s_m^S}$ is intersecting, and therefore, the family of (crisp) edges *E(H)* is intersecting as well.

Example 3. Consider an IFDHG H with

Here $V = \{v_1\, v_2, v_3, v_4\}, E = \{E_1\, E_2, E_3\}$ and whose incidence matrix is as follows:

$$H = \begin{matrix} v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \end{matrix} \begin{pmatrix} E_1 & E_2 & E_3 \\ \langle 0.7,0.2\rangle & \langle 0.7,0.2\rangle & \langle 0.7,0.2\rangle \\ \langle 0,1\rangle & \langle 0.5,0.2\rangle & \langle 0.5,0.2\rangle \\ \langle 0,1\rangle & \langle 0.5,0.4\rangle & \langle 0.5,0.4\rangle \\ \langle 0.3,0.2\rangle & \langle 0,1\rangle & \langle 0.3,0.2\rangle \end{pmatrix}$$

Clearly, $h(H) = \langle 0.7, 0.2\rangle$. Then,

$E^{0.7,0.2} = \{\{v_1, v_4\}\}$

$E^{0.5,0.2} = \{\{v_1, v_4\}, \{v_1, v_2, v_3\}\}$

$E^{0.5,0.4} = \{\{v_1, v_4\}, \{v_1, v_2, v_3\}\}$

$E^{0.3,0.2} = \{\{v_1, v_4\}, \{v_1, v_2, v_3\}, \{v_1, v_2, v_3, v_4\}\}$

Thus, $0.3 < r \leq 0.7$ and $0.2 \leq s \leq 0.4$

$E^{r,s} = \{v_1\} = E^{0.7,0.2}$

and for $0 < r \leq 0.3$ and $0.4 \leq s < 1$

$E^{r,s} = \{\{v_1, v_4\}, \{v_1, v_2, v_3\}, \{v_1, v_2, v_3, v_4\}\} = E^{0.3,0.2}$

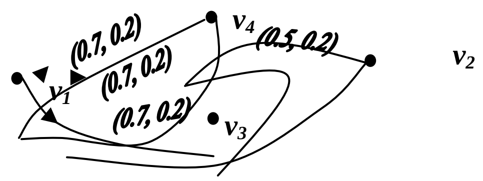

Figure. 4: K− intersecting IFDHG

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Note that, $E^{0.7,0.2} \subset E^{0.3,0.2}$ Therefore, $E^{0.7,0.2} \subset E^{0.5,0.2} \subset E^{0.5,0.4} \subset E^{0.3,0.2}$. Thus, H is an ordered intuitionistic fuzzy directed hypergraph.

$H^{0.7,0.2} = (V_1, E_1) = \{\{v_1, v_4\}\}$

$H^{0.5,0.2} = (V_2, E_2) = \{\{v_1, v_4\}, \{v_1, v_2, v_3\}\}$

$H^{0.5,0.4} = (V_3, E_3) = \{\{v_1, v_4\}, \{v_1, v_2, v_3\}\}$

$H^{0.3,0.2} = (V_4, E_4) = \{\{v_1, v_4\}, \{v_1, v_2, v_3\}, \{v_1, v_2, v_3, v_4\}\}$

Thus H is a K-intersecting intuitionistic fuzzy directed hypergraph.

Example 4. Consider an IFDHG $H = (V, E)$ where $V = \{v_1, v_2, v_3\}$ and $E = \{E_1\, E_2, E_3\}$ which is represented by the following adjacency matrix:

$$H = \begin{matrix} & E_1 & E_2 & E_3 \\ v_1 & \langle 0.6,0.4\rangle & \langle 0,1\rangle & \langle 0.6,0.4\rangle \\ v_2 & \langle 0.4,0.3\rangle & \langle 0.4,0.3\rangle & \langle 0,1\rangle \\ v_3 & \langle 0.5,0.2\rangle & \langle 0.5,0.2\rangle & \langle 0.5,0.2\rangle \end{matrix}$$

Clearly, $h(H) = \langle 0.6, 0.4\rangle$

$E^{0.6,0.4} = \{\{v_1\}\}$

$E^{0.5,0.2} = \{\{v_1, v_3\}\}$

$E^{0.4,0.3} = \{\{v_1, v_2, v_3\}, \{v_2, v_3\}, \{v_1, v_3\}\}$

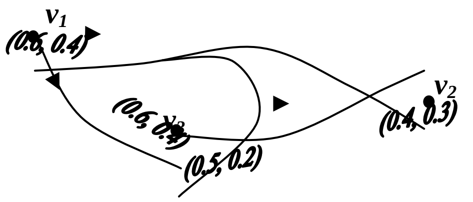

Figure. 5: Non-ordered intersecting IFDHG

Therefore, $E^{0.6,0.4} \sqsubset E^{0.5,0.2} \sqsubset E^{0.4,0.3}$ but, $E^{0.6,0.4} \not\subseteq E^{0.5,0.2}$. Hence, H is non - ordered.

$H^{0.6,0.4} = (V_1, E_1) = (\{v_1\}, \{\{v_1\}\})$

$H^{0.5,0.2} = (V_2, E_2) = (\{v_1, v_3\}, \{\{v_3\}, \{v_1, v_3\}\})$

$H^{0.4,0.3} = (V_3, E_3) =$

$(\{v_1, v_2, v_3\}, \{\{v_1, v_2, v_3\}, \{v_2, v_3\}, \{v_3, v_1\}\})$

Thus H is non K-intersecting IFDHG.

IV. CONCLUSION

In this paper, an attempt has been made to study the chromatic values and chromatic numbers of intuitionistic fuzzy hypergraph colorings. Also, upper and lower truncation, core aggregate, conservative *K*-coloring, intersecting, *K*-intersecting, strongly intersecting IFDHGs were studied. Also, elementary center, *f*-chromatic value of intuitionistic fuzzy coloring are discussed. It has been proved that an IFDHG is strongly intersecting if and only if it is *K*-intersecting. If *H* is an ordered intersecting IFDHG, then each intuitionistic fuzzy hyperedge *T* of *H* contains a member of $Tr(H_{h(T)})$.

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