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Some Studies on Energy of Triple Connected Graphs

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***Abstract:** The field of mathematics plays a vital role in various fields, one of the important areas in mathematics is graph theory. The concept of connectedness plays an important role in any networks.Let G be a simple graph with n vertices and m edges. The ordinary energy of a graph is defined as sum of absolute values of eigen values of its adjacency matrix. In recent times analogous of energies are being considered based on eigen values of variety of other graph matrices. In this paper we analyzed various energies of triple connected graphs and obtained bounds.*

***Keywords:** energy, eigen values, triple connected graphs, incidence energy, AMS Mathematics Subject Classification (2010): 05C78*

I. INTRODUCTION

In mathematics_graph theory is the study of graphs, which are mathematical structures used to model pairwise relations between objects. A graph in this context is made up of vertices**,** nodes, or points which are connected by edges arcs and lines. A graph may be undirected, meaning that there is no distinction between the two vertices associated with each edge, or its edges may be directed from one vertex to another. A graph G consists of a pair (V,E) where V is a non-empty finite set whose elements are called vertices or nodes or points and E is another set whose elements are called edges or lines such that each edge is associated with ordered or unordered pair of elements. Energy of different graphs including regular, no regular, circulant and random graphs is also under study.

$E_C(G)$ denotes Color energy of the graph

$E_L(G)$ denotes Laplacian energy of graph

$IE(G)$ denotes Incidence energy of graph

II. PRELIMINARIES

Definition: 1[1]

A graph is said to be triple connected if any three vertices lie on a path in G.

A triple connected graph may or may not have cut vertex or cut edge.

Definition:2 [2]

A energy of the graph is represented by sum of the absolute values of the eigen values of the graph G and is denoted by E(G).Thus E(G)=∑|λi|,i=1 to n.

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Definition: 3[7]

The laplacian matrix of graph L(G) is defined as L(G)=D(G)-A(G) where D(G) is the degree of the matrix and A(G) is adjacency matrix of G.

Laplacian matrix can be used to calculate the no. of spanning trees for a given graph

$$L_{i,j} = \begin{cases} d_i & if i = j \\ -1 & if\ i\ is adjacent to j, i \neq j \\ 0 & otherwise \end{cases}$$

Definition: 4 [3]

A coloring of graph G is a coloring of its vertices such that no two adjacent vertices receive the same color.

For vertex colored graph, entries of the matrix are as follows .If $C(V_i)$ is the color of V_i, then

$$a_{i,j} = \begin{cases} 1, & if\ v_i\ and\ v_j\ are\ adjacent \\ -1, & if\ v_i\ and\ v_j\ are\ nonadjacent\ with\ C(v_1) = C(v_2) \\ 0, & otherwise \end{cases}$$

Definition: 5 [5]

The incidence matrix I(G) is an n-by-m matrix for which

$$I(G) = \begin{cases} 1, & if\ \ v\ is\ incidence\ with\ edge C_{j_i} \\ 0, & otherwise \end{cases}$$

Definition 6: [7]

Given a simple graph *G* with *n* vertices, its Laplacian matrix L_{n*n} is defined as:

$L = D - A$, where *D* is the degree matrix and *A* is the adjacency matrix of the graph. Since G is a simple graph, A only contains 1's or 0's and its diagonal elements are all 0's. In the case of directed graphs, either the indegree or outdegree might be used, depending on the application. The elements of L are given by:

$$L_{i,j} = \begin{cases} \deg(v_i), & if\ i = j \\ -1, & if\ i \neq j \\ 0, & otherwise \end{cases}$$

III. RESULTS AND OBSERVATIONS

A. Some Bounds for Paths:

Preposition 3.1: If G is a triple connected graph then

$$radius(G) = \begin{cases} 1 & if\ n = 3 \\ 2 & if\ n = 4 \\ 3 & if\ n \geq 5 \end{cases}$$ and

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$$diam(G) = n-1 \quad if \quad n \geq 3$$

Preposition 3.2: If G is a triple connected graph and $G^{'}$ is its complement then $E(G) \leq E(G^{'})$

Preposition 3.3 If G is a triple connected graph then $E_c(G) \leq E_L(G)$

B. Some Bounds for Cycle

Preposition 3.4: If G is a cycle then $radius(G) \leq n - \Delta$ for n ≥ 3

Preposition 3.5: If G is cycle then the incidence energy is always less than the complement of that $IE(G) \leq IE(G^{'})$

C. Some Bounds for Compete Graph and Complete Bipartite Graph:

Preposition 3. 6: If G is complete graph then $radius(G) = diam(G) = 1$

Preposition 3.7: If G is complete graph then $E(G) = E_C(G)$

Preposition 3.8: If G is complete bipartite graph then $radius(G) = diam(G) = 2$

For $K_{2,2}$ E(G)=4

For $K_{2,3}$ E(G)=4.8

For $K_{3,2}$ E(G)=4.8

For $K_{3,3}$ E(G)=6

Hence from the above result we can conclude the E ($K_{m,n}$)=E ($K_{n,m}$)

D. Bounds for Wheel Graphs:

Preposition 3.9: If G is a wheel graph, then $radius(G) = 1 \quad for \ n \geq 4$ and

$$diam(G) = \begin{cases} 1 & if \ n = 4 \\ 2 & if \ n \geq 5 \end{cases}$$

Preposition 3.10: If G is a wheel graph for n≥4, then $E(G) \leq E_C(G) \leq E_L(G)$.

E. Bounds for Peterson Graph

If G is a Peterson graph, then $E(G) = 6$ and $E(G) \leq E_C(G) \leq E_L(G)$

F. Bounds for Directed Cyces

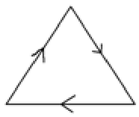
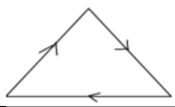
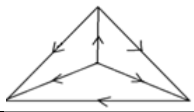
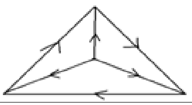
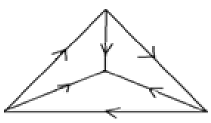
Preposition 1.1 If G is a directed cycle, then

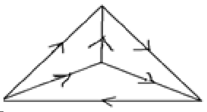
$$radius(G) \leq n - \Delta \quad and \quad diam(G) \leq n - \Delta \quad for \ n \geq 3$$

$$E(G) = \begin{cases} 3 & if \ n = 3 \\ 4 & if \ n = 4 \\ 5 & if \ n = 5 \end{cases}$$

The above result for energy of G is taken when out deg=1 and in deg=0 in the adjacency matrix

G. Energy of Tournaments in Digraphs:

Tournaments	Energy of digraph with Indeg=0 Outdeg=1	Energy of digraph with Indeg= -1 Outdeg=1	Max indeg	Max outdeg
	3. 000	3.4641	1	1
	0	2.8284	2	2
	0	5.6569	3	3
	3.000	6.9282	2	3
	3.000	6.9282	3	2

	4.8677	5.6569	2	2

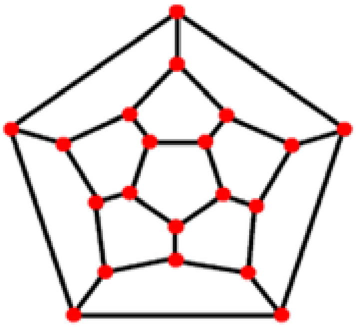
From the above table we note that 2 tournaments (n=4) whose max indeg=max outdeg have same energy (when indeg=-1 and outdeg=1 in adjacency matrix).

If G is a strong oriented graph, then E(G) increases with increase the value of n.we have analysed in two ways how the enrgy values differ if we take indeg is 0 and indegree is -1

H. Bounds for Strong Oriented Graphs:

n-no. of vertices	Energy Indeg=0 Outdeg=1	Energy Indeg= -1 Outdeg=1	Max indeg	Max outdeg
3	3.000	3.4641	1	1
4	4.0719	4.9274	2	2
5	6.4721	7.6085	2	2

I. Dodechedran Graph:



Energy =29.4164

IV. CONCLUSION

In this paper we have analayzed the value of different kinds of energy in triple connected graphs and digraphs also given some basic bounds of it.In our future work is to find the detailed proof of all bounds.

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