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System Reliability using Generalized Intuitionistic Fuzzy Exponential Lifetime Distribution

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***Abstract:** The aim of this paper is investigates the reliability characteristics of systems using generalized intuitionistic fuzzy exponential lifetime distribution, in which the lifetime parameter is assumed to be generalized intuitionistic fuzzy number. Generalized intuitionistic fuzzy reliability, generalized intuitionistic fuzzy hazard function, generalized intuitionistic fuzzy mean time to failure and their (α_1, α_2)-cut have been discussed when systems follow generalized intuitionistic fuzzy exponential lifetime distribution. Further, reliability analysis of the series and parallel systems has been done.*

***Index Terms:** generalized intuitionistic fuzzy number (GIFN), (α_1, α_2) -cut, generalized intuitionistic fuzzy distribution, generalized intuitionistic fuzzy reliability.*

I. INTRODUCTION

After the introduction of fuzzy sets by Zadeh (1965), many researchers used the concept of fuzzy set to deal with uncertainty in reliability analysis. For example: Singer (1990), Cia et al. (1993), Chen (1994), Mong et al. (1994), Pandey and Tyagi (2007), Pandey et al. (2009), Baloui Jamkhaneh (2011), Baloui Jamkhaneh (2014) and etc. Atanassov (1986) introduced concept intuitionistic fuzzy sets (IFS) as a generalization of fuzzy sets. Intuitionistic fuzzy sets theory is a useful tool in modeling real life problems, wherein hesitation between belongingness and non-belongingness cannot be ruled out. Burillo et al. (1994) proposed the definition of intuitionistic fuzzy number (IFN). Mahapatra and Roy (2009) presented triangular intuitionistic fuzzy number (TIFN) and used it for reliability evaluation. Mahapatra and Mahapatra (2010) presented intuitionistic fuzzy fault tree using arithmetic operation of trapezoidal intuitionistic fuzzy number (TrIFN) which are evaluated based on (α,β)-cuts method. Pandey et al. (2011) describes a novel approach, based on intuitionistic fuzzy set theory for reliability analysis of series and parallel network. Kumar et al. (2011) developed a new approach for analyzing the fuzzy system reliability of series and parallel systems using intuitionistic fuzzy set theory. Kumar and Yadav (2011) presented a new method for fuzzy system reliability analysis based on arithmetic operations of different types of intuitionistic fuzzy numbers. Sharma (2012) presented the reliability of a system using IFS. Garg et al. (2013) predicted the behavior of an industrial system under imprecise and vague environment. To handle the uncertainty in the data, they used IFS theory rather than fuzzy set theory. Also Garg and Rani (2013) presented a technique for computing the membership functions of the intuitionistic fuzzy set (IFS) in reliability analyzed by utilizing imprecise, uncertain and vague data. Bohra and singh (2015) used intuitionistic fuzzy Rayleigh distribution in evaluating the systems reliability. Kumar and Singh (2015) presented fuzzy system reliability using intuitionistic fuzzy Weibull lifetime distribution. Kumar and Singh (2017) investigated the applications of generalized trapezoidal intuitionistic fuzzy number in fuzzy reliability theory.

Baloui Jamkhaneh and Nadarajah (2015) considered new generalized intuitionistic fuzzy sets ($GIFS_B$) and introduced some operators over $GIFS_B$. Shabani and Baloui Jamkhaneh (2014) introduced a new generalized intuitionistic fuzzy number ($GIFN_B$) based on generalization of the IFS. The main objective of this paper is to evaluated systems reliability using generalized intuitionistic fuzzy exponential distribution, in which the parameter of the exponential distribution is taken as a generalized intuitionistic fuzzy number related to Shabani and Baloui Jamkhaneh (2014). The originality of this study comes from the fact that, there were no previous works in generalized intuitionistic fuzzy distributions. This paper is organized as follows: In Section 2, we briefly introduce generalized intuitionistic fuzzy numbers. In Section 3 define generalized intuitionistic fuzzy distribution. In Section 4, generalized intuitionistic fuzzy reliability is introduced. In Section 5, generalized intuitionistic fuzzy reliability of series and parallel system is calculated. The paper is concluded in Section 6.

II. PRELIMINARIES

A. Generalized Intuitionistic Fuzzy Number

Definition1. (Baloui Jamkhaneh, Nadarajah (2015)) Let X be a non empty set. A generalized intuitionistic fuzzy sets ($GIFS_B(X)$) A in X, is defined as an object of the form $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle : x \in X\}$ where the functions $\mu_A : X \to [0,1]$ and $\nu_A : X \to [0,1]$, denote the degree of membership and degree of non membership functions of A respectively, and $0 \le \mu_A(x)^\delta + \nu_A(x)^\delta \le 1$ for each $x \in X$ and $\delta = n \text{ or } \frac{1}{n}, n = 1,2,\dots,N$.

Definition2. (Shabani, Baloui Jamkhaneh (2014)) In special case, generalized L-R type intuitionistic fuzzy number A can be described as any $GIFS_B(X)$ of the real line ℝ whose membership function $\mu_A(x)$ and non-membership function $\nu_A(x)$ are defined as follows

$$\mu_A(x) = \begin{cases} (\frac{x-a}{b-a})^{\frac{1}{\delta}} & , \quad a \le x \le b \\ 1 & , \quad b \le x \le c \\ (\frac{d-x}{d-c})^{\frac{1}{\delta}} & , \quad c \le x \le d \\ 0 & , \quad o.w \end{cases},$$

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$$\nu_A(x)=\begin{cases}(\frac{b-x}{b-a_1})^{\frac{1}{\delta}} & , \quad a_1\le x\le b\\ 0 & , \quad b\le x\le c\\ (\frac{x-c}{d_1-c})^{\frac{1}{\delta}} & , \quad c\le x\le d_1\\ 1 & , \quad o.w\end{cases},$$

where $a_1\le a\le b\le c\le d\le d_1$ and $0\le \mu_A(x)^\delta+\nu_A(x)^\delta\le 1, \forall x\in X$. The $GIFN_B$ A is denoted as A = (a1,a,b,c,d,d1,δ).

Definition3. A $GIFN_B$ is said to be symmetric $GIFN_B$ if $b-a=d-c$ and $b-a_1=d_1-c$.

B. Cut Sets on $GIFN_B$

Let $\alpha_1,\alpha_2\in[0,1]$ be fixed numbers such that $0\le\alpha_1\le 1, 0\le\alpha_2\le 1, 0\le{\alpha_1}^\delta+{\alpha_2}^\delta\le 1$. A set of (α_1,α_2)-cut generated by a $GIFN_B$ A is defined by

$$A[\alpha_1,\alpha_2,\delta]=\{\langle x,\mu_A(x)\ge\alpha_1,\nu_A(x)\le\alpha_2\rangle : x\in X\},$$

$A[\alpha_1,\alpha_2,\delta]$ is defined as the crisp set of elements x which belong to A at least to the degree α_1 and which does not belong to A at most to the degree α_2. A α_1- cut set of a $GIFN_B$ A is a crisp subset of $\mathbb{R}$, which defined is as

$$A[\alpha_1,\delta]=\{\langle x,\mu_A(x)\ge\alpha_1,\rangle : x\in X\} \quad , \quad 0\le\alpha_1\le 1.$$

According to the definition of $GIFN_B$, it can be easily shown that

$$A[\alpha_1,\delta]=[L_1(\alpha_1),U_1(\alpha_1)] \quad , \quad 0\le\alpha_1\le 1,$$
$$L_1(\alpha_1)=a+(b-a){\alpha_1}^\delta \quad , \quad U_1(\alpha)=d-(d-c){\alpha_1}^\delta.$$

Similarity a α_2- cut set of a $GIFN_B$ A is a crisp subset of $\mathbb{R}$, which defined is as

$$A[\alpha_2,\delta]=\{\langle x,\nu_A(x)\le\alpha_2\rangle : x\in X\} \quad , \quad 0\le\alpha_2\le 1.$$

According to the definition of $GIFN_B$it can be easily shown that

$$A[\alpha_2,\delta]=[L_2(\alpha_2),U_2(\alpha_2)] \quad , \quad 0\le\alpha_2\le 1,$$

$$L_2(\alpha_2)=b(1-{\alpha_2}^\delta)+a_1{\alpha_2}^\delta,$$
$$U_2(\beta)=c(1-{\alpha_2}^\delta)+d_1{\alpha_2}^\delta.$$

Therefore the (α_1,α_2)-cut of a $GIFN_B$is given by

$$A[\alpha_1,\alpha_2,\delta]=\{x, x\in[L_1(\alpha_1),U_1(\alpha_1)]\cap[L_2(\alpha_2),U_2(\alpha_2)]\}.$$

III. GENERALIZED INTUITIONISTIC FUZZY DISTRIBUTION

Let the lifetime of a component (X) is modeled by $f(x,\lambda)$, where λ is a $GIFN_B$. In this case, the generalized intuitionitic fuzzy probability of obtaining a value in the interval [n,m] is as $\widetilde{P}(n\le X\le m)$ and compute its cut sets as follows

$$P(n\le X\le m)[\alpha_i]=\{\textstyle\int_n^m f(x,\lambda)dx|\lambda\in\lambda[\alpha_i,\delta]\},$$
$$=[P^L[\alpha_i],P^U[\alpha_i]], \quad i=1,2$$

for all $0\le\alpha_1\le 1, 0\le\alpha_2\le 1, 0\le{\alpha_1}^\delta+{\alpha_2}^\delta\le 1$, where

$$P^L[\alpha_i]=\min\{\textstyle\int_n^m f(x,\lambda)dx|\lambda\in\lambda[\alpha_i,\delta]\},$$
$$P^U[\alpha_i]=\max\{\textstyle\int_n^m f(x,\lambda)dx|\lambda\in\lambda[\alpha_i,\delta]\},$$

with this definition, $\widetilde{P}(n\le X\le m)$ is a $GIFN_B$, where $[P^L[\alpha_1],P^U[\alpha_1]]$ and $[P^L[\alpha_2],P^U[\alpha_2]]$ are α_1- cut set of membership function and α_2- cut set of non-membership function respectively. Also (α_1,α_2)- cut set

$$P(n\le X\le m)[\alpha_1,\alpha_2]=$$
$$[P^L[\alpha_1],P^U[\alpha_1]]\cap[P^L[\alpha_2],P^U[\alpha_2]].$$

Let lifetime of a component (X) is modeled by an exponential distribution, then

$$f(x,\lambda)=\lambda e^{-\lambda x}, \quad x>0, \qquad \lambda>0,$$

where $\tilde{\lambda}$ is a generalized intuitionistic fuzzy number. In this case we have

$$\widetilde{P}(n\le X\le m)[\alpha_i]=\{\int_n^m \lambda e^{-\lambda x}dx\,|\lambda\in\lambda[\alpha_i,\delta]\}$$
$$=[P^L[\alpha_i],P^U[\alpha_i]], \quad i=1,2$$

for all $0\le\alpha_1\le 1, 0\le\alpha_2\le 1, 0\le{\alpha_1}^\delta+{\alpha_2}^\delta\le 1$, where

$$P^L[\alpha_i]=\min\{\textstyle\int_n^m \lambda e^{-\lambda x}dx\,|\lambda\in\lambda[\alpha_i,\delta]\},$$
$$=\min\{e^{-\lambda n}-e^{-\lambda m}|\lambda\in\lambda[\alpha_i,\delta]\},$$
$$P^U[\alpha_i]=\max\{\textstyle\int_n^m \lambda e^{-\lambda x}dx\,|\lambda\in\lambda[\alpha_i,\delta]\},$$
$$=\max\{e^{-\lambda n}-e^{-\lambda m}|\lambda\in\lambda[\alpha_i,\delta]\}.$$

IV. RELIABILITY CHARACTERISTICS

A. Generalized Intuitionistic Fuzzy Reliability

Reliability function (S(t)) is the probability a unit survives beyond time t. Let the random variable X denote lifetime of a system component, also let X has lifetime density function $f(x,\lambda)$, and cumulative distribution function $F_X(t)=P(X\le t)$, and then the reliability function at time t is defined as

$$S(t)=P(X>t)=1-F(t), \quad t>0.$$

Suppose that we want to calculate reliability of component, such that the lifetime variable has exponential distribution with generalized intuitionistic fuzzy lifetime parameter $\tilde{\lambda}=(a_1,a,b,c,d,d_1,\delta)$. In this case, generalized intuitionistic fuzzy reliability (GIFR) of component is as $\tilde{S}(t)$ and compute its cut sets is as follows

$$S(t)[\alpha_i]=\{\textstyle\int_t^\infty f(x,\lambda)dx\,|\lambda\in\lambda[\alpha_i,\delta]\},$$
$$=\{\textstyle\int_t^\infty \lambda e^{-\lambda x}dx\,|\lambda\in\lambda[\alpha_i,\delta]\},$$
$$=\{e^{-\lambda t}|\lambda\in\lambda[\alpha_i,\delta]\}, \quad i=1,2.$$

Since $e^{-\lambda t}$ is a monotonically decreasing function, then α_1- cut set of membership function and α_2 - cut set of non-membership function are as follows

$$S(t)[\alpha_1] = [S^L[\alpha_1], S^U[\alpha_1]], \\ = \left[e^{-\left(d-(d-c)\alpha_1^\delta\right)t}, e^{-\left(a+(b-a)\alpha_1^\delta\right)t}\right],$$

$$S(t)[\alpha_2] = [S^L[\alpha_2], S^U[\alpha_2]], \\ = \left[e^{-\left(c\left(1-\alpha_2^\delta\right)+d_1\alpha_2^\delta\right)t}, e^{-\left(b\left(1-\alpha_2^\delta\right)+a_1\alpha_2^\delta\right)t}\right],$$

$S(t)[\alpha_i], i = 1,2$ are two dimensional function in terms of α_i and t ($0 \le \alpha_1 \le 1, 0 \le \alpha_2 \le 1, 0 \le {\alpha_1}^\delta + {\alpha_2}^\delta \le 1$ and $t > 0$). For t_0, $\tilde{S}(t_0)$ is a generalized intuitionistic fuzzy number and membership function and non-membership function of $\tilde{S}(t_0)$ are as follows

$$\mu_{S(t_0)}(x) = \begin{cases} \left(\frac{d+\frac{\ln x}{t_0}}{d-c}\right)^{\frac{1}{\delta}}, & e^{-dt_0} \le x \le e^{-ct_0} \\ 1, & e^{-ct_0} \le x \le e^{-bt_0} \\ \left(\frac{-a-\frac{\ln x}{t_0}}{b-a}\right)^{\frac{1}{\delta}}, & e^{-bt_0} \le x \le e^{-at_0} \\ 0, & o.w. \end{cases},$$

$$\nu_{S(t_0)}(x) = \begin{cases} \left(\frac{-c-\frac{\ln x}{t_0}}{d_1-c}\right)^{\frac{1}{\delta}}, & e^{-d_1t_0} \le x \le e^{-ct_0} \\ 0, & e^{-ct_0} \le x \le e^{-bt_0} \\ \left(\frac{b+\frac{\ln x}{t_0}}{b-a_1}\right)^{\frac{1}{\delta}}, & e^{-bt_0} \le x \le e^{-a_1t_0} \\ 1, & o.w. \end{cases},$$

A (α_1, α_2)- cut set of $\tilde{S}(t)$ is as follows
$S(t)[\alpha_1, \alpha_2] = S(t)[\alpha_1] \cap S(t)[\alpha_2]$.
In this method, for every especially α_1 and α_2 (as α_{10} and α_{20}) reliability curve is like a band with upper and lower bounds. In this case, it is called GIFR band. This reliability band has properties follows:
(i) $S(0)[\alpha_{10}, \alpha_{20}] = 1$, i.e. no one starts off dead,
(ii) $S(\infty)[\alpha_{10}, \alpha_{20}] = 0$, i.e. everyone dies eventually,
(iii) $S(t_1)[\alpha_{10}, \alpha_{20}] \ge S(t_2)[\alpha_{10}, \alpha_{20}] \Leftrightarrow t_1 \le t_2$, i.e. band of $S(t)[\alpha_{10}, \alpha_{20}]$ declines monotonically.

Corollary1. Let $\eta = \frac{\alpha_2^\delta}{1-\alpha_1^\delta}$, $k_1 = \frac{b-a}{b-a_1}$ and $k_2 = \frac{d-c}{d_1-c}$, then we have
if $k_1 < k_2$

$$S(t_0)[\alpha_1, \alpha_2] = \begin{cases} [S^L[\alpha_2], S^U[\alpha_2]], & \eta < k_1 \\ [S^L[\alpha_2], S^U[\alpha_1]], & k_1 \le \eta \le k_2 \\ [S^L[\alpha_1], S^U[\alpha_1]], & k_2 < \eta \end{cases}$$

if $k_1 > k_2$

$$S(t_0)[\alpha_1, \alpha_2] = \begin{cases} [S^L[\alpha_2], S^U[\alpha_2]], & \eta < k_2 \\ [S^L[\alpha_1], S^U[\alpha_2]], & k_2 \le \eta \le k_1 \\ [S^L[\alpha_1], S^U[\alpha_1]], & k_1 < \eta \end{cases}$$

if $k_1 = k_2 = k$ (i.e. λ is symmetric $GIFN_B$)

$$S(t_0)[\alpha_1, \alpha_2] = \begin{cases} [S^L[\alpha_2], S^U[\alpha_2]], & \eta < k \\ [S^L[\alpha_1], S^U[\alpha_1]], & \eta \ge k \end{cases}$$

If $\eta = 1$ (i.e. $\alpha_2^\delta = 1 - \alpha_1^\delta$), then

$$S(t_0)[\alpha_1, \alpha_2] = [S^L[\alpha_1], S^U[\alpha_1]].$$

B. Generalized Intuitionistic Fuzzy Hazard Function

Generalized intuitionistic fuzzy hazard function (GIFHF) of component is as follows

$$h(t)[\alpha_i] = \left\{\frac{f(t)}{s(t)} | \lambda \in \lambda[\alpha_i, \delta]\right\}, \quad i = 1,2,$$

for all $0 \le \alpha_1 \le 1, 0 \le \alpha_2 \le 1, 0 \le {\alpha_1}^\delta + {\alpha_2}^\delta \le 1$, where

$$h(t)^L[\alpha_i] = \min\{\tfrac{f(t)}{s(t)} | \lambda \in \lambda[\alpha_i, \delta]\},$$
$$h(t)^U[\alpha_i] = \max\{\tfrac{f(t)}{s(t)} | \lambda \in \lambda[\alpha_i, \delta]\}, \ i = 1,2 .$$

A (α_1, α_2)- cut set of $\tilde{h}(t)$ is as follows

$$h(t)[\alpha_1, \alpha_2] = h(t)[\alpha_1] \cap h(t)[\alpha_2],$$

$h(t)[\alpha_i], i = 1,2$ are two dimensional function in terms of α_i and t. For t_0, $\tilde{h}(t_0)$ is a generalized intuitionistic fuzzy number. In this method, for every especially α_1 and α_2 (as α_{10} and α_{20}) hazard function ($\tilde{h}(t)$) is like a band with upper and lower bounds. In this case, it is called GIFH band. When lifetime variable has generalized intuitionistic fuzzy exponential distribution, then

$$h(t)[\alpha_i] = \{\lambda | \lambda \in \lambda[\alpha_i, \delta]\}, \quad i = 1,2,$$

$$h(t)[\alpha_1] = [a + (b-a){\alpha_1}^\delta, d - (d-c){\alpha_1}^\delta] ,$$
$$h(t)[\alpha_2] = [b\left(1 - {\alpha_2}^\delta\right) + a_1{\alpha_2}^\delta, c\left(1 - {\alpha_2}^\delta\right) + d_1{\alpha_2}^\delta] .$$

Thus, for a generalized intuitionistic fuzzy exponential distribution, the hazard function is a constant with respect to time.

Corollary2. Let $\eta = \frac{\alpha_2^\delta}{1-\alpha_1^\delta}$, $k_1 = \frac{b-a}{b-a_1}$ and $k_2 = \frac{d-c}{d_1-c}$, then we have
if $k_1 < k_2$

$$h(t_0)[\alpha_1, \alpha_2] = \begin{cases} [h^L[\alpha_2], h^U[\alpha_2]], & \eta < k_1 \\ [h^L[\alpha_1], h^U[\alpha_2]], & k_1 \le \eta \le k_2 \\ [h^L[\alpha_1], h^U[\alpha_1]], & k_2 < \eta \end{cases},$$

if $k_1 > k_2$

$$h(t_0)[\alpha_1, \alpha_2] = \begin{cases} [h^L[\alpha_2], h^U[\alpha_2]], & \eta < k_2 \\ [h^L[\alpha_2], h^U[\alpha_1]], & k_2 \le \eta \le k_1 \\ [h^L[\alpha_1], h^U[\alpha_1]], & k_1 < \eta \end{cases},$$

if $k_1 = k_2 = k$

$$h(t_0)[\alpha_1, \alpha_2] = \begin{cases} [h^L[\alpha_2], h^U[\alpha_2]], & \eta < k \\ [h^L[\alpha_1], h^U[\alpha_1]], & \eta \ge k \end{cases},$$

If $\eta = 1$, then $h(t_0)[\alpha_1, \alpha_2] = [h^L[\alpha_1], h^U[\alpha_1]]$.

C. Generalized Intuitionistic Fuzzy Mean Time to Failure

Generalized intuitionistic fuzzy mean time to failure (GIFMTTF) is the expected the mean time to failure.

According definition, MTTF of any generalized intuitionistic fuzzy component is a generalized intuitionistic fuzzy number and can be defined as follows

$GIFMTTF[\alpha_i] = \{\int_0^{\infty} s(x)dx \,|\lambda \in \lambda[\alpha_i, \delta]\}, \quad i = 1,2,$

when the lifetimes have generalized intuitionistic fuzzy exponential distribution then

$$\begin{aligned} GIFMTTF[\alpha_i] &= \{\int_0^{\infty} s(x)dx \,|\lambda \in \lambda[\alpha_i, \delta]\}, \\ &= \{\int_0^{\infty} e^{-\lambda t}dx \,|\lambda \in \lambda[\alpha_i, \delta]\}, \\ &= \{\tfrac{1}{\lambda} |\lambda \in \lambda[\alpha_i, \delta]\}, \quad i = 1,2, \end{aligned}$$

$$GIFMTTF[\alpha_1] = \left[\frac{1}{d-(d-c)\alpha_1^{\delta}}, \frac{1}{(a+(b-a)\alpha_1^{\delta})}\right],$$

$$GIFMTTF[\alpha_2] = \left[\frac{1}{(c(1-\alpha_2^{\delta})+d_1\alpha_2^{\delta})}, \frac{1}{(b(1-\alpha_2^{\delta})+a_1\alpha_2^{\delta})}\right],$$

membership function and non-membership function of GIFMTTF is as follows

$$\mu_G(x) = \begin{cases} \left(\frac{d-\frac{1}{x}}{d-c}\right)^{\frac{1}{\delta}} , & \frac{1}{d} \le x \le \frac{1}{c} \\ 1 , & \frac{1}{c} \le x \le \frac{1}{b} \\ \left(\frac{\frac{1}{x}-a}{b-a}\right)^{\frac{1}{\delta}} , & \frac{1}{b} \le x \le \frac{1}{a} \\ 0 , & o.w. \end{cases},$$

$$\nu_G(x) = \begin{cases} \left(\frac{\frac{1}{x}-c}{d_1-c}\right)^{\frac{1}{\delta}} , & \frac{1}{d_1} \le x \le \frac{1}{c} \\ 0 , & \frac{1}{c} \le x \le \frac{1}{b} \\ \left(\frac{b-\frac{1}{x}}{b-a_1}\right)^{\frac{1}{\delta}} , & \frac{1}{b} \le x \le \frac{1}{a_1} \\ 1 , & o.w \end{cases}.$$

V. NUMERICAL EXAMPLE

Let lifetime of electronic component is modeled by an exponential distribution with generalized intuitionistic fuzzy parameter $\tilde{\lambda} = (0.3,0.35,0.4,0.45,0.5,0.55,2)$. Then α_i-cut of generalized intuitionistic fuzzy probability of $0 \le X \le 1$ is as follows

$\tilde{P}(0 \le X \le 1)[\alpha_i] = \{1 - e^{-\lambda} |\lambda \in \lambda[\alpha_i, 2]\}, \quad i = 1,2$

$\tilde{P}(0 \le X \le 1)[\alpha_1] =$
$[1 - e^{-(0.35+0.05\alpha_1^2)}, 1 - e^{-(0.5-0.05\alpha_1^2)}],$
$\tilde{P}(0 \le X \le 1)[\alpha_2] = [1 - e^{-(0.4-0.1\alpha_2^2)}, 1 - e^{-(0.45+0.1\alpha_2^2)}].$

The membership function and non-membership function of $\tilde{P}(0 \le X \le 1)$ is as follows

$$\mu_P(x) = \begin{cases} \left(\frac{-0.35-\ln(1-x)}{0.05}\right)^{\frac{1}{2}} , & 1 - e^{-0.35} \le x \le 1 - e^{-0.4} \\ 1 , & 1 - e^{-0.4} \le x \le 1 - e^{-0.45} \\ \left(\frac{0.5+\ln(1-x)}{0.05}\right)^{\frac{1}{2}} , & 1 - e^{-0.45} \le x \le 1 - e^{-0.5} \\ 0 , & o.w. \end{cases},$$

$$\nu_P(x) = \begin{cases} \left(\frac{0.4+\ln(1-x)}{0.1}\right)^{\frac{1}{2}} , & 1 - e^{-0.3} \le x \le 1 - e^{-0.4} \\ 0 , & 1 - e^{-0.4} \le x \le 1 - e^{-0.45} \\ \left(\frac{-0.45-\ln(1-x)}{0.1}\right)^{\frac{1}{2}} , & 1 - e^{-0.45} \le x \le 1 - e^{-0.55} \\ 1 , & o.w. \end{cases}.$$

Table1. Values of $\tilde{P}(0 \le X \le 1)$ for different (α_1, α_2) -cut

α_1	α_2	$\tilde{P}(0 \le X \le 1)$
1	0	$[1 - e^{-0.4}, 1 - e^{-0.45}]$
0.5	0.5	$[1 - e^{-0.375}, 1 - e^{-0.475}]$
0	1	$[1 - e^{-0.35}, 1 - e^{-0.5}]$

α_i-cut and (α_1, α_2) -cut of GIFR of component is given by

$$\begin{aligned} S(t)[\alpha_1] &= \left[e^{-(0.5-0.05\alpha_1^2)t}, e^{-(0.35+0.05\alpha_1^2)t}\right], S(t)[\alpha_2], \\ &= \left[e^{-(0.45+0.1\alpha_2^2)t}, e^{-(0.4-0.1\alpha_2^2)t}\right], \end{aligned}$$

$S(t)[\alpha_1, \alpha_2] = S(t)[\alpha_1] \cap S(t)[\alpha_2].$

If $t = 1$ then membership and non-membership function of GIFR are as

$$\begin{aligned} S(1)[\alpha_1] &= \left[e^{-(0.5-0.05\alpha_1^2)}, e^{-(0.35+0.05\alpha_1^2)}\right], S(1)[\alpha_2], \\ &= \left[e^{-(0.45-0.1\alpha_2^2)}, e^{-(0.4-0.1\alpha_2^2)}\right], \end{aligned}$$

The membership function and non-membership function of $\tilde{S}(1)$ is as follows

$$\mu_{S(1)}(x) = \begin{cases} \left(\frac{0.5+\ln x}{0.05}\right)^{\frac{1}{2}} , & e^{-0.5} \le x \le e^{-0.45} \\ 1 , & e^{-0.45} \le x \le e^{-0.4} \\ \left(\frac{-0.35-\ln x}{0.05}\right)^{\frac{1}{2}} , & e^{-0.4} \le x \le e^{-0.35} \\ 0 , & o.w. \end{cases},$$

$$\nu_{S(1)}(x) = \begin{cases} \left(\frac{-0.45-\ln x}{0.1}\right)^{\frac{1}{2}} , & e^{-0.55} \le x \le e^{-0.45} \\ 0 , & e^{-0.45} \le x \le e^{-0.4} \\ \left(\frac{0.4+\ln x}{0.1}\right)^{\frac{1}{2}} , & e^{-0.4} \le x \le e^{-0.3} \\ 1 , & o.w. \end{cases},$$

A (0.5,0.5)- cut set of $\tilde{S}(t)$ is as follows

$S(t)[\alpha_1] = [e^{-0.4875t}, e^{-0.3625t}],$

$S(t)[\alpha_2] = [e^{-0.475t}, e^{-0.375t}],$

$S(t)[0.5,0.5] = S(t)[\alpha_1] \cap S(t)[\alpha_2] = [e^{-0.475t}, e^{-0.375t}],$

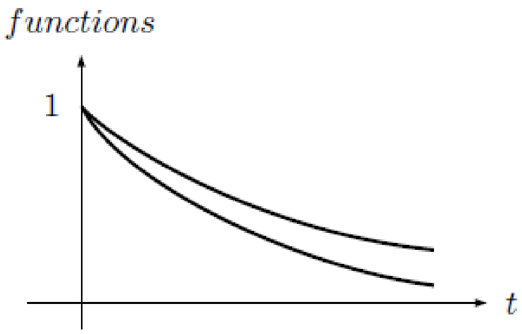

Fig.1: Generalized intuitionistic fuzzy reliability band for $\boldsymbol{\alpha_1 = 1\,, \alpha_2 = 0}$

α_i-cut and (α_1, α_2) -cut of GIFHF is given by

$$\mathrm{h(t)}[\alpha_1] = [0.35 + 0.05\alpha_1^2, 0.5 - 0.05\alpha_1^2],$$

$$\mathrm{h(t)}[\alpha_2] = [0.4 - 0.1\alpha_2^2, 0.45 + 0.1\alpha_2^2],$$

The membership function and non-membership function of $\tilde{\mathrm{h}}(\mathrm{t})$ is as follows

$$\mu_{\mathrm{h(t)}}(\mathrm{x}) = \begin{cases} \left(\frac{\mathrm{x}-0.35}{0.05}\right)^{\frac{1}{2}} & , \quad 0.35 \le \mathrm{x} \le 0.4 \\ 1 & , \quad 0.4 \le \mathrm{x} \le 0.45 \\ \left(\frac{0.5-\mathrm{x}}{0.05}\right)^{\frac{1}{2}} & , \quad 0.45 \le \mathrm{x} \le 0.5 \\ 0 & , \quad \mathrm{o.w} \end{cases},$$

$$\nu_{\mathrm{h(t)}}(\mathrm{x}) = \begin{cases} \left(\frac{0.4-\mathrm{x}}{0.1}\right)^{\frac{1}{2}} & , \quad 0.3 \le \mathrm{x} \le 0.4 \\ 0 & , \quad 0.4 \le \mathrm{x} \le 0.45 \\ \left(\frac{-0.45+\mathrm{x}}{0.1}\right)^{\frac{1}{2}} & , \quad 0.45 \le \mathrm{x} \le 0.55 \\ 1 & , \quad \mathrm{o.w} \end{cases}$$

A (1,0)- cut set of $\tilde{\mathrm{h}}(\mathrm{t})$ is as follows: $\mathrm{h(t)}(1,0) = [0.4, 0.45]$

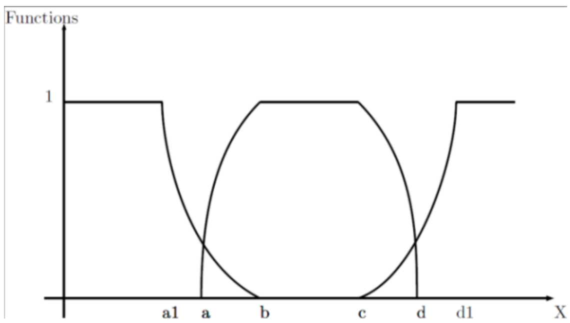

Fig.2: Membership and non-membership functions of $\tilde{\mathbf{h}}(\mathbf{t})$

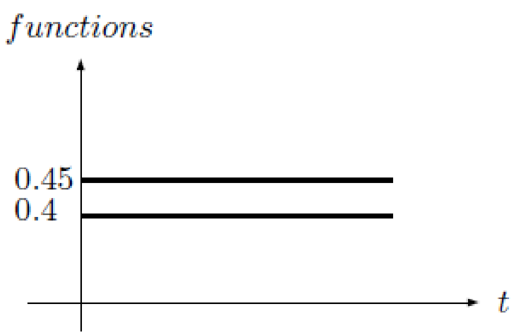

Fig.3: Generalized intuitionistic fuzzy hazard band for $\boldsymbol{\alpha_1 = 1\,, \alpha_2 = 0}$

α_i-cut of GIFMTTF is given by

$$\mathrm{GIFMTTF}[\alpha_1] = \left[\frac{1}{0.5-0.05\alpha_1^2}, \frac{1}{0.35+0.05\alpha_1^2}\right],$$
$$\mathrm{GIFMTTF}[\alpha_2] = \left[\frac{1}{0.45+0.1\alpha_2^2}, \frac{1}{0.4-0.1\alpha_2^2}\right].$$

Membership function and non-membership function of GIFMTTF is as follows

$$\mu_{\mathrm{G}}(\mathrm{x}) = \begin{cases} \left(\frac{0.5-\frac{1}{\mathrm{x}}}{0.05}\right)^{\frac{1}{2}} & , \quad \frac{1}{0.5} \le \mathrm{x} \le \frac{1}{0.45} \\ 1 & , \quad \frac{1}{0.45} \le \mathrm{x} \le \frac{1}{0.4} \\ \left(\frac{\frac{1}{\mathrm{x}}-0.35}{0.05}\right)^{\frac{1}{2}} & , \quad \frac{1}{0.4} \le \mathrm{x} \le \frac{1}{0.35} \\ 0 & , \quad \mathrm{o.w} \end{cases},$$

$$\nu_{\mathrm{G}}(\mathrm{x}) = \begin{cases} \left(\frac{\frac{1}{\mathrm{x}}-0.45}{0.1}\right)^{\frac{1}{2}} & , \quad \frac{1}{0.55} \le \mathrm{x} \le \frac{1}{0.45} \\ 0 & , \quad \frac{1}{0.45} \le \mathrm{x} \le \frac{1}{0.4} \\ \left(\frac{0.4-\frac{1}{\mathrm{x}}}{0.1}\right)^{\frac{1}{2}} & , \quad \frac{1}{0.4} \le \mathrm{x} \le \frac{1}{0.3} \\ 1 & , \quad \mathrm{o.w} \end{cases},$$

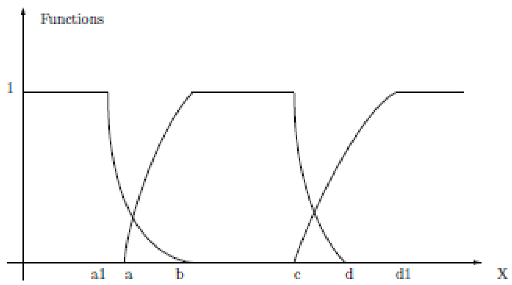

Fig.4: Membership and non-membership functions of GIFMTTF

VI. GIFR OF SERIES AND PARALLEL SYSTEM

In this section, we have evolved a generalized intuitionistic fuzzy reliability evaluation technique for series and parallel systems.

A. Series System

If *n*-components are connected in series, then the α_i-cut $(i = 1,2)$ of GIFR with generalized intuitionistic fuzzy exponential distribution is given by

$$\mathrm{S(t)}[\alpha_i] = \{\mathrm{P}(\mathrm{Y}_1 > t)\ |\lambda \in \lambda[\alpha_i, \delta]\},$$
$$= \{\mathrm{e}^{-\mathrm{n}\lambda\mathrm{t}} |\lambda \in \lambda[\alpha_i, \delta]\}, \quad i = 1,2,$$

$$\mathrm{S(t)}[\alpha_1] = \left[\mathrm{e}^{-\mathrm{n}\left(\mathrm{d}-(d-c)\alpha_1^{\delta}\right)t}, \mathrm{e}^{-\mathrm{n}\left(\mathrm{a}+(b-a)\alpha_1^{\delta}\right)t}\right],$$

$$\mathrm{S(t)}[\alpha_2] = \left[\mathrm{e}^{-\mathrm{n}\left(c\left(1-\alpha_2^{\delta}\right)+d_1\alpha_2^{\delta}\right)t}, \mathrm{e}^{-\mathrm{n}\left(b\left(1-\alpha_2^{\delta}\right)+a_1\alpha_2^{\delta}\right)t}\right].$$

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For t_0, GIFR is a generalized intuitionistic fuzzy number and membership function and non-membership function of $\tilde{S}(t)$is as follows

$$\mu_{S(t_0)}(x)=\begin{cases}\left(\frac{d+\frac{\ln x}{nt_0}}{d-c}\right)^{\frac{1}{\delta}} & , \quad e^{-ndt_0}\le x\le e^{-nct_0}\\ 1 & , \quad e^{-nct_0}\le x\le e^{-nbt_0}\\ \left(\frac{-a-\frac{\ln x}{nt_0}}{b-a}\right)^{\frac{1}{\delta}} & , \quad e^{-nbt_0}\le x\le e^{-nat_0}\\ 0 & , \quad o.w\end{cases},$$

$$\nu_{S(t_0)}(x)=\begin{cases}\left(\frac{-c-\frac{\ln x}{nt_0}}{d_1-c}\right)^{\frac{1}{\delta}} & , \quad e^{-nd_1t_0}\le x\le e^{-nct_0}\\ 0 & , \quad e^{-nct_0}\le x\le e^{-nbt_0}\\ \left(\frac{b+\frac{\ln x}{nt_0}}{b-a_1}\right)^{\frac{1}{\delta}} & , \quad e^{-nbt_0}\le x\le e^{-na_1t_0}\\ 1 & , \quad o.w.\end{cases}.$$

B. Parallel system

If n-components are connected in parallel, then the $\boldsymbol{\alpha_i}$-cut ($i=1,2$) of GIFR with intuitionistic fuzzy exponential distribution is given by

$$\begin{aligned}S(t)[\alpha_i]&=\{P(Y_n>t)\,|\lambda\in\lambda[\alpha_i,\delta]\},\\ &=\{1-(1-e^{-\lambda t})^n|\lambda\in\lambda[\alpha_i,\delta]\},\quad i=1,2\end{aligned}$$

$$\begin{aligned}S(t)[\alpha_1]=\Big[1-\left(1-e^{-\left(d-(d-c)\alpha_1^{\delta}\right)t}\right)^n,1\\ -(1-e^{-\left(a+(b-a)\alpha_1^{\delta}\right)t})^n\Big],\end{aligned}$$

$$\begin{aligned}S(t)[\alpha_2]=\Big[1-(1-e^{-\left(c\left(1-\alpha_2^{\delta}\right)+d_1\alpha_2^{\delta}\right)t})^n),1\\ -(1-e^{-n\left(b\left(1-\alpha_2^{\delta}\right)+a_1\alpha_2^{\delta}\right)t})^n\Big].\end{aligned}$$

For t_0, this is a generalized intuitionistic fuzzy number and membership function and non-membership function of $\tilde{S}(t_0)$is as follows

$$\mu_{S(t_0)}(x)=\begin{cases}\left(\frac{t_0d+\ln(1-(1-x)^{\frac{1}{n}})}{t_0(d-c)}\right)^{\frac{1}{\delta}} & , \quad 1-(1-e^{-dt_0})^n\le x\le 1-(1-e^{-ct_0})^n\\ 1 & , \quad 1-(1-e^{-ct_0})^n\le x\le 1-(1-e^{-bt_0})^n\\ \left(\frac{-at_0-\ln(1-(1-x)^{\frac{1}{n}})}{t_0(b-a)}\right)^{\frac{1}{\delta}} & , \quad 1-\left(1-e^{-bt_0}\right)^n\le x\le 1-(1-e^{-at_0})^n\\ 0 & , \quad o.w.\end{cases},$$

$$\nu_{S(t_0)}(x)=\begin{cases}\left(\frac{-ct_0-\ln\left(1-(1-x)^{\frac{1}{n}}\right)}{t_0(d_1-c)}\right)^{\frac{1}{\delta}} & , \quad 1-\left(1-e^{-d_1t_0}\right)^n\le x\le 1-(1-e^{-ct_0})^n\\ 0 & , \quad 1-(1-e^{-ct_0})^n\le x\le 1-\left(1-e^{-bt_0}\right)^n.\\ \left(\frac{bt_0+\ln(1-(1-x)^{\frac{1}{n}})}{t_0(b-a_1)}\right)^{\frac{1}{\delta}} & , \quad 1-\left(1-e^{-bt_0}\right)^n\le x\le 1-(1-e^{-a_1t_0})^n\\ 1 & , \quad o.w\end{cases}$$

VII. CONCLUSION

The generalized intuitionistic fuzzy reliability function and generalized intuitionistic fuzzy hazard function have been successfully investigated in this paper. Intuitionistic fuzzy system reliability is based on the concept of intuitionistic fuzzy set and intuitionistic fuzzy probability theory in our method. In this paper the parameter of lifetime distribution has been taken $GIFN_B$. In this approach, $S(t)[\alpha_i]$ is a two dimensional function in terms of α_i and t. For t_0, $\tilde{S}(t_0)$ and $\tilde{h}(t_0)$ are generalized intuitionistic fuzzy numbers and for every especially α_1 and α_2, reliability curve and hazard curve are like a band. Finally we described reliability analysis of series and parallel system base on generalized intuitionistic fuzzy lifetime parameter. Our method is more comprehensive than previous methods. Since, if $\delta=1$ then the results agree with intuitionistic fuzzy reliability evaluation, if $\delta=1$ and $\alpha_1^{\delta}=1-\alpha_2^{\delta}$, it changes to intuitionistic fuzzy reliability evaluation duo to Bohra and Singh (2015) and Kumar and Singh (2015), if $\delta=1$, $\alpha_1^{\delta}=1-\alpha_2^{\delta}$, $a=a_1$ and $d=d_1$, it changes to fuzzyreliability evaluation duo to Baloui Jamkhaneh (2011, 2014).

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AUTHORS PROFILE

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