Pitch Mode Control System Design of Guided Missile

Muhammad A. R Yass, Mohammed Wajeeh Ameen

***Abstract:** In this paper, the analysis of non-homogenous longitudinal equation of motion for transport Airplane in pitch mode to estimate and calculate the performance of the dynamic motion and Transfer function in pitch mode. The pitch feedback control system diagram with actuator Design to calculate the behavior using time response method for different gain values (K) and Different Gyro sensitivity values (GR) to obtain the best stability, peak value and time response. The results shows that the best stability and control behavior achieved when K=1.41 and GR= 1.19.*

***Keyword:** control system, stability, aerodynamic*

I. INTRODUCTION

Pitch –up is most likely to occur in airplane that have the horizontal stabilizer mounted well above the airplane, common place is the top of the vertical stabilizer[1]. This is sometime done to obtain the end –plate effect on the vertical stabilizer and thus increase effectiveness of the Vertical [2] .Another factor that contributes to this unstable flight condition is the wing a low aspect ratio .such a using has large downwash velocity that increase rapidly as the angle of attack of the wing is increased .as the high horizontal tail moves down into this wake, pitch – up occurs if the downwash velocity becomes high enough. The longitudinal feedback control system was design to hold the airplane in straight and level fight .there is no simple rule to aid.[3]

Engineer in selecting final sensitivity .the final choose usually results in a compromise between the desirability of rapid response and desire to reduce excessive overshot the ability of prediction the systems performance by an analysis that does required the actual solution of The differential equation .also we would like to indicate this analysis reading the manner or the method by which this system must be adjusted compensated to produce the desired performance characteristics[4].

Two basic methods available to choose analyze and interpret the state suicidal response of the transfer function of the system obtains an idea of the systems response .this method is based upon the interpretation of SyQuest plot. Although this frequency response approach doesn't yield an exact quantitative prediction of the systems performance i.e. the poles of the control ratio C(s)/r(s) cannot be determine enough information can be obtained to indicate whether the system need to be the system should be compensate [5].

Revised Version Manuscript Received on August 08, 2017

Dr. Muhammad A. R Yass, Assistant Professor, Department of Electromechanical Engineering, University of Technology, Baghdad 00964, Iraq. E-mail: Mohd.yass97@gmail.com

Mohammed Wajeeh Ameen, MSc., Department of Computer Techs. Engineering, Dijlah University College, Baghdad 00964, Iraq. E-mail: baba_19620@yahoo.com

II. MATHEMATICAL ANALYSIS

The values of the stability derivatives for a four engine jet transport are used. The aircraft is flying in straight and level flight at 40,000 ft with a velocity of 600 ft per sec (355 knots), and the compressibility effects will be neglected [6]. The three longitudinal equations were derived in reference to q and as given below: (dynamic notes)

$$\frac{mv}{sq}i-\frac{d}{2U}cm_{\alpha}\alpha+\frac{Iy}{sqd}\theta-\frac{d}{2U}cmq\theta^{\circ}=CFxa\text{..............}(1)$$

$$-(C2u\overline{U})+\left[\frac{mu}{sq}-\frac{d}{2u}C_2x)\alpha-C_{2\alpha}\alpha\right]$$

$$+\left[(\frac{mu}{sq}-\frac{d}{2u}C_2q)\theta^{\circ}-\ddot{C}_w(\sin(H))\theta\right]=CF_{2a}\text{............}(2)$$

$$(-C_{mo}{}'V+(\frac{-d}{2U}C_{m\alpha}\alpha-C_{m\alpha}\alpha)+(\frac{Iy}{Sqd}\ddot{\theta}-\frac{d}{2U}C_{mq}{}^{\circ}e)=cma\text{........}(3)$$

$$(13.78s+0.088)'_u(s)-0.392'_\alpha(s)+0.7\theta(s)=0\ \ldots\ldots\ (4)$$

$$1.48'_u(s)-(13.78s+4.46)'_\alpha(s)-13.78s\theta(s)=0\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ (5)$$

$$0+(0.0552s+0.619)'_\alpha(s)+(0.514s^2+0.192s)\theta(s)=0\ \ldots\ldots\ldots\ldots\ (6)$$

The only nonzero solution of these simultaneous equations requires that the determinate of the coefficient be zero[8]. Thus

$$\begin{vmatrix}13.78s+0.088 & -0.392 & 0.74\\ 1.48 & 13.78s+4.46 & -13.78s\\ 0 & 0.0552s+0.619 & 0.514s^2+0.192s\end{vmatrix}=0$$

Expanding this determinant, the following quartic equation is obtained [7].

$$9754s^4+79s^3+128.9s^2+0.998s+0.677=0\ \ldots\ldots\ldots\ (7)$$

Dividing through by **975**, the equation reduces to

$$s^4+0.811s^3+1.32s^2+0.012s+0.00695=0\ \ldots\ldots\ldots\ldots\ (8)$$

*Published By:
Blue Eyes Intelligence Engineering
& Sciences Publication*

The transfer function for 6, input to $'\alpha$ output in determinant form is

$$\frac{'_{\alpha}(S)}{\delta_e} = \frac{\begin{vmatrix} 13.78s + 0.088 & -0.392 & 0.74 \\ 1.48 & -0.246 & -13.78s \\ 0 & -0.710 & 0.514s^2 + 0.192s \end{vmatrix}}{\nabla}$$

$$\frac{'_{\alpha}(S)}{\delta_e}$$

$$= \frac{-0.01785(s^3 + 77.8s^2 + 0.496s + 0.446)}{s^2 + 0.00466s + 0.0053)(s^2 + 0.806s + 1.311)} \dots\dots\dots\dots.(9)$$

Factoring the numerator

$$\frac{'_{\alpha}(S)}{\delta_e} = \frac{-0.01785(s + 77.79)(s^2 + 0.0063s + 0.0057)}{s^2 + 0.00466s + 0.0053)(s^2 + 0.806s + 1.311)} \dots\dots\dots\dots..\dots\dots..(10)$$

Going to the alternate form, Eq. [10] becomes

$$\frac{'_{\alpha}(S)}{\delta_e} = \frac{-1.14\left(\frac{s}{77.79} + 1\right)\left[\left(\frac{s}{0.0755}\right)^2 + \frac{2(0.041)}{0.0755}s + 1\right]}{\left[\left(\frac{s}{0.073}\right)^2 + \frac{2(0.032)}{0.073}s + 1\right]\left[\left(\frac{s}{1.14}\right)^2 + \frac{2(0.352)}{1.145}s + 1\right]} \dots\dots\dots...(11)$$

For input to θ output

$$\frac{\theta(S)}{\delta_e} = \frac{\begin{vmatrix} 13.78s + 0.088 & -0.392 & 0 \\ 1.48 & 13.78s + 4.46 & -0.246 \\ 0 & 0.0552s + 0.619 & -0.710 \end{vmatrix}}{\nabla}$$

Expanding and factoring

$$\frac{\theta(S)}{\delta_e} = \frac{-1.31(s + 0.016) + (s + 0.3)}{s^2 + 0.00466s + 0.0053)(s^2 + 0.806s + 1.311)} \dots\dots\dots\dots..\dots\dots..(12)$$

Or the alternate form

$$\frac{\theta(S)}{\delta_e(s)} = \frac{-0.95\left(\frac{s}{0.016} + 1\right)\left(\frac{s}{0.3} + 1\right)}{\left[\left(\frac{s}{0.073}\right)^2 + \frac{2(0.032)}{0.073}s + 1\right]\left[\left(\frac{s}{1.145}\right)^2 + \frac{2(0.352)}{1.145}s + 1\right]} \dots\dots\dots\dots..(13)$$

Substituting for q and simplifying, Eq. [10] becomes

$$\zeta = -\frac{1}{4}\left(C_{mq} + C_{m\alpha} + \frac{2I_y}{Cm^2}C_{z\alpha}\right)\left(\frac{cm^2}{I_y\left(\frac{C_{mq}\,C_{z\alpha}}{2} - \frac{2mC_{mq}}{\rho Sc}\right)}\right)^{1/2} \dots\dots\dots\dots\dots\dots\dots.(14)$$

$$\frac{'_{\alpha}(S)}{\delta_e} = \frac{\begin{vmatrix} -.246 & -13.78s \\ -0.710 & 0.514s^2 + 0.192s \end{vmatrix}}{\begin{vmatrix} 13.78s + 4.45 & -13.78s \\ 0.0552s + 0.619 & 0.514s^2 + 0.192s \end{vmatrix}} \dots\dots\dots.\dots\dots\dots\dots\dots\dots\dots\dots.\dots(15)$$

Expanding

$$\frac{'_{\alpha}(S)}{\delta_e(s)} = \frac{-0.01782(s + 77.8)}{s^2 + 0.805s + 1.3325} \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots..\dots\dots..(16)$$

Going to the alternate form

$$\frac{'_{\alpha}(S)}{\delta_e(s)} = \frac{-1.05\left(\frac{s}{77.8} + 1\right)}{\left(\frac{s}{1.15}\right)^2 + \frac{2(0.35)}{1.15}s + 1} \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots...\dots(17)$$

A comparison of Eq. [10] and [12] shows excellent agreement substantiating the original assumption. The $\theta(s)/\delta e(s)$ transferfunction will now be evaluated.

$$\frac{\theta(S)}{\delta_e(s)} = \frac{\begin{vmatrix} 13.78s + 4.46 & -0.246 \\ 0.0552s + 0.619 & -0.710 \end{vmatrix}}{\begin{vmatrix} 13.78s + 4.45 & -13.78s \\ 0.0552s + 0.619 & 0.514s^2 + 0.192s \end{vmatrix}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (18)$$

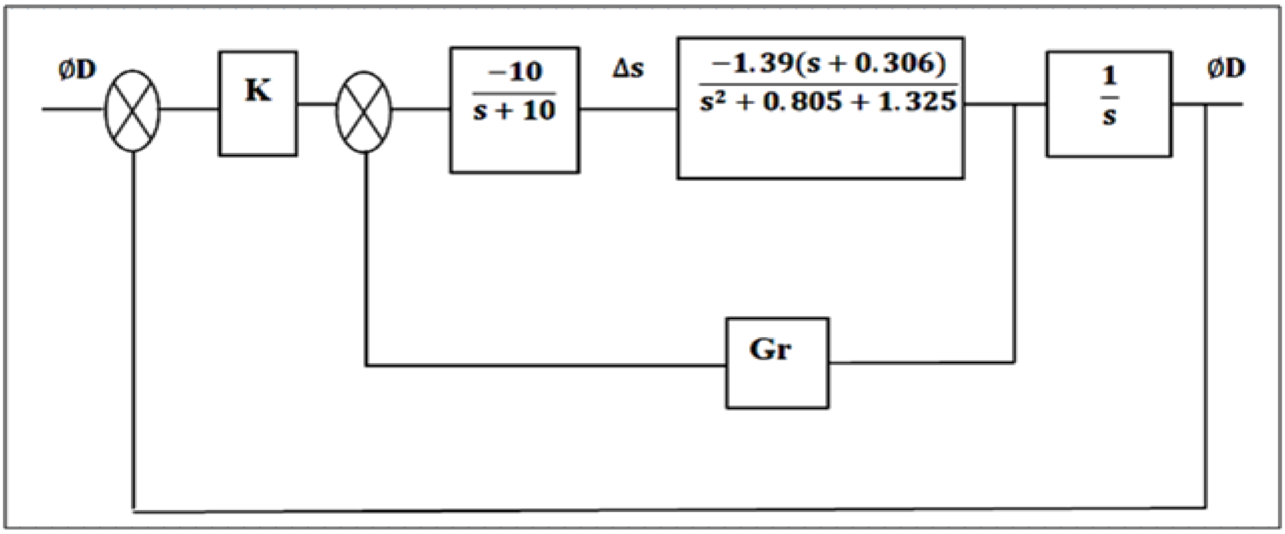
Expanding,

$$\frac{\theta(S)}{\delta_e(s)} = \frac{-1.39(s + 0.306)}{s(s^2 + 0.805s + 1.3325)} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (19)$$

Going to the alternate form

$$\frac{\theta(S)}{\delta_e(s)} = \frac{-0.321\left(\frac{s}{0.306} + 1\right)}{s\left[\left(\frac{s}{1.15}\right)^2 + \frac{2(0.35)}{1.15}s + 1\right]} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (20)$$

The magnitude plot of Eq. [18] is shown in Figure (2-8).



$$\frac{\theta(S)}{\delta_e(s)} = \frac{13.9k(s + 0.306)}{s^5 + 10.805s^4 + (9.375 + 13.9GR)s^3 + (13.255 + 4.253Gr + 13.9k)s^2 + 4.254ks}$$

$$F(x) = \frac{19.6x+6}{x^5+10.805x^4+25.16x^3+37.196x^2+6x}$$

$$F(x) = \frac{1}{x} + \frac{-0.24071+0.3223i}{x+1.215-1.62075i} + \frac{-0.24071-0.3223i}{x+1.215+1.62075i} + \frac{-0.0458}{x+8.1967} + \frac{-0.47276}{x+0.178408}$$

Q=1-de^{-zt}-ce^{-pt}+2e^{-gt}(-a cosht+b sinht)

a=0.24071 g=1.1215 z=8.12 c=0.4727

b=0.3223 h=1.6207 p=0.178 d=0.0458

III. RESULTS AND DISSECTION

3.1. Longitudinal Response at Different K&GR:

Table (1):

GR	K	Characteristics Equations	Real Root	Imaginary Root	Steady Time	Max Peak Over Shoot	stability
1	1	s^5+10.805s^4+23.725s^3+31.4s^2+4.25s	0.15	1.06±1.52	13	1.25	stable
1	1.1	s^5+10.805s^4+23.725+32.8s^2+4.68s	0.156	1.04±1.57	13	1.3	stable
1	1.2	s^5+10.805s^4+23.725+34.$s^2$2+5.1s	0.16	1.01±1.62	12.5	1.4	stable

Published By:
Blue Eyes Intelligence Engineering
& Sciences Publication

1	1.3	s^5+10.805s^4+23.725+35.58s^2+5.53s	0.17	1±1.67	15	1.475	stable
1	1.4	s^5+10.805s^4+23.725+36.96s^2+5.95s	0.18	0.904±1.69	10.3	1.48	stable
1	1.6	s^5+10.805s^4+23.725+38.96s^2+	0.185	0.909±1.801	10	1.56	stable
1	1.7	s^5+10.805s^4+23.725+39.74s^2+6.23	0.189	0.92±1.832	11	1.6	stable
1	1.8	s^5+10.805s^4+23.725+41.52s^2+7.6	0.193	0.928±1.86	10	1.65	stable

Table (2):

GR	K	Characteristics Equations	Real Root	Imaginary Root	Steady Time	Max Peak Over Shoot	stability
1.3	1	s^5+10.805s^4+27.445+32.6s^2+4.2s	0.14	1.3±1.42	16	1.25	stable
1.3	1.1	s^5+10.805s^4+27.445+35.4s^2+4.62s	0.16	1.3±1.43	10	2	unstable
1.3	1.2	s^5+10.805s^4+27.445+36.7s^2+5.1s	1.161	1.3±1.48	9	1.35	stable
1.3	1.3	s^5+10.805s^4+27.445+38.4s^2+5.5s	0.17	1.36±1.49	9.8	0.75	unstable
1.3	1.4	s^5+10.805s^4+27.445+39.6s^2+5.95s	0.175	1.54±1.57	11	0.512	unstable
1.3	1.5	s^5+10.805s^4+27.445+41s^2+6.3s	0.18	1.32±1.63	9	0.8	unstable

Table (3):

GR	K	Characteristics Equations	Real Root	Imaginary Root	Steady Time	Max Peak Over Shoot	stability
1.2	1	s^5+10.805s^4+26.005+31.8s^2+4.25s	0.14	1.3±1.36	15	1.23	stable
1.2	1.1	s^5+10.805s^4+26.005+34.6s^2+5.1s	1.15	1.27±1.43	15	1.32	stable
1.2	1.2	s^5+10.805s^4+26.005+36.7s^2+5.5s	1.16	1.26±1.5	15	1.37	stable
1.2	1.3	s^5+10.805s^4+26.005+37.4s^2+5.95s	1.172	1.24±1.55	15	1.4	stable
1.2	1.4	s^5+10.805s^4+26.005+38.9s^2+6.3s	1.174	1.2±1.59	10	1.42	stable
1.2	1.5	s^5+10.805s^4+26.005+40.2s^2+6.8s	1.176	1.23±1.59	11	1.52	stable

Table (4)

GR	K	Characteristics Equations	Real Root	Imaginary Root	Steady Time	Max Peak Over Shoot	stability
1.1	1	s^5+10.805s^4+24.66+31.83s^2+4.25s	0.157	1.18±1.48	14	1.25	stable
1.1	1.1	s^5+10.805s^4+24.66+32.6s^2+4.6s	0.159	1.178±1.499	14	1.3	stable
1.1	1.2	s^5+10.805s^4+24.66+36.5s^2+5.1s	0.161	1.1±1.51	13.9	1.4	stable
1.1	1.3	s^5+10.805s^4+24.66+37.6$s^2$5+5.5s	0.1637	1.098±1.598	15	1.43	stable
1.1	1.4	0.1705s^5+10.805s^4+24.66+38.2s^2+ 5.95s	0.169	1.078±1.63	11	1.42	stable
1.1	1.5	s^5+10.805s^4+24.66+40.1s^2+6.3s	0.172	1.0678±1.78	12	1.51	stable
1.1	1.6	s^5+10.805s^4+24.66+41.07s^2+6.8s	0.186	1.063±1.803	12	1.56	stable
1.1	1.7	s^5+10.805s^4+24.66+42.93s^2+7.23s	0.189	1.059±1.81	15	1.59	stable

Table (5):

GR	K	Characteristics Equations	Real Root	Imaginary Root	Steady Time	Max Peak Over Shoot	stability
1.4	1	s^5+10.805s^4+28.83+33.1s^2+4.2s	0.146	1.59±1.2	9	1.21	stable
1.4	1.1	s^5+10.805s^4+28.83+33.4s^2+4.6s	0.15	1.52±1.22	9	1.22	stable
1.4	1.2	s^5+10.805s^4+28.83+35.7s^2+5.1s	0.16	1.52±1.21	10	1.298	stable
1.4	1.3	s^5+10.805s^4+28.83+37.2s^2+5.5s	0.161	1.5±1.35	9.8	1.32	stable
1.4	1.5	s^5+10.805s^4+28.83+38.6v+5.95s	0.17	1.48±1.43	11	1.4	stable
1.4	1.6	s^5+10.805s^4+28.83+40s^2+6.3s	0.18	1.46±1.5	11	1.4	stable
1.4	1.7	s^5+10.805s^4+28.83+41.4s^2+6.8s	0.18	1.44±1.56	9	1.478	unstable
1.4	1.8	s^5+10.805s^4+28.83+42.8s^2+7.23s	0.193	1.42±1.62	9	1.52	stable

Table (6):

GR	K	Characteristics Equations	Real Root	Imaginary Root	Steady Time	Max Peak Over Shoot	stability
1.5	1	s^5+10.805s^4+30.225+33.53s^2+4.25s	0.143	1.402±1.52	15	1.22	stable
1.5	1.1	s^5+10.805s^4+30.225+34.9s^2+4.67s	0.15	1.41±1.53	15	1.288	stable
1.5	1.2	s^5+10.805s^4+30.225+36.3v+5.1s	0.162	1.42±1.55	14	1.3	stable
1.5	1.3	s^5+10.805s^4+30.225+37.7s^2+5.5s	0.164	1.43±1.61	13.5	1.33	stable
1.5	1.4	s^5+10.805s^4+30.225+39s^2+5.95s	0.17	1.439±1.72	14	1.83	stable
1.5	1.5	s^5+10.805s^4+30.225+40.59s^2+6.38s	0.178	1.51±1.799	13	1.41	stable

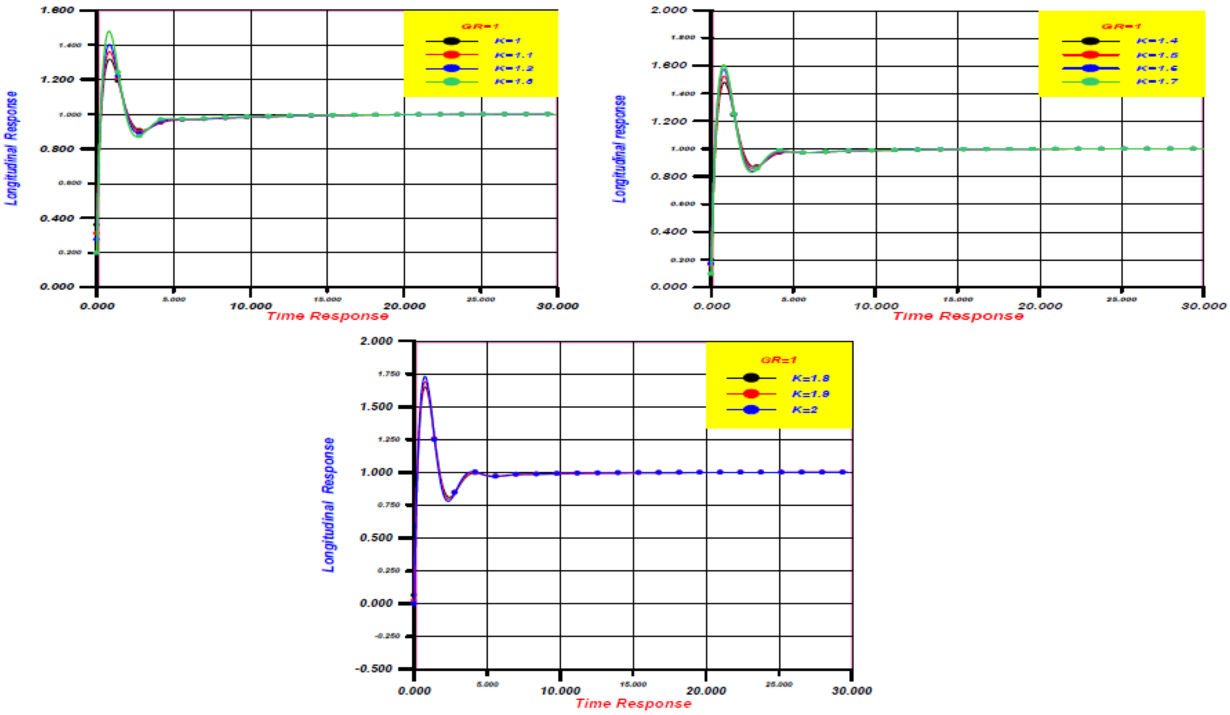


Figure (2) Longitudinal Response verses Time response for GR=1and Different Gain Value

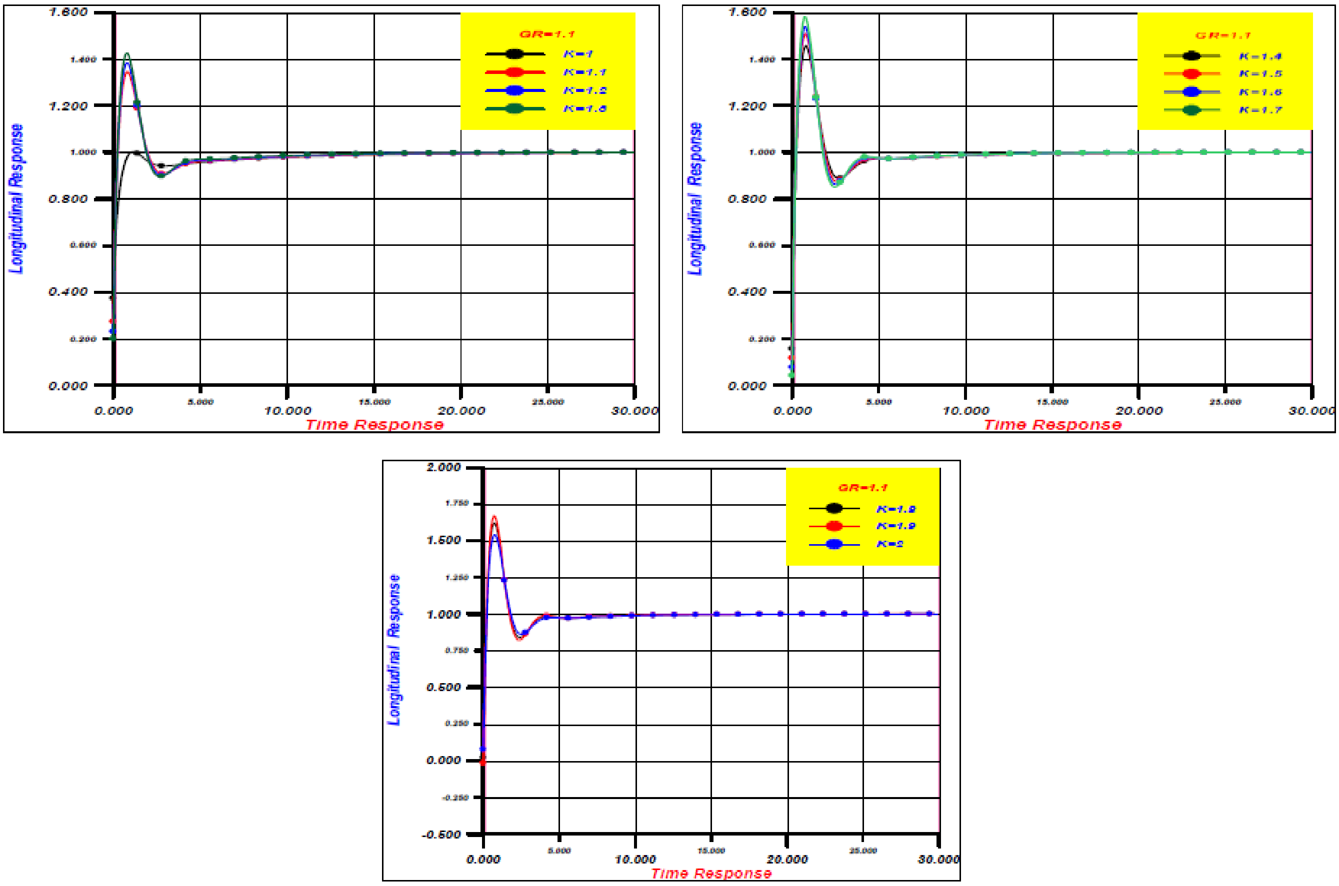


Figure (3) Longitudinal Response verses Time response for GR=1.1 and Different Gain Value

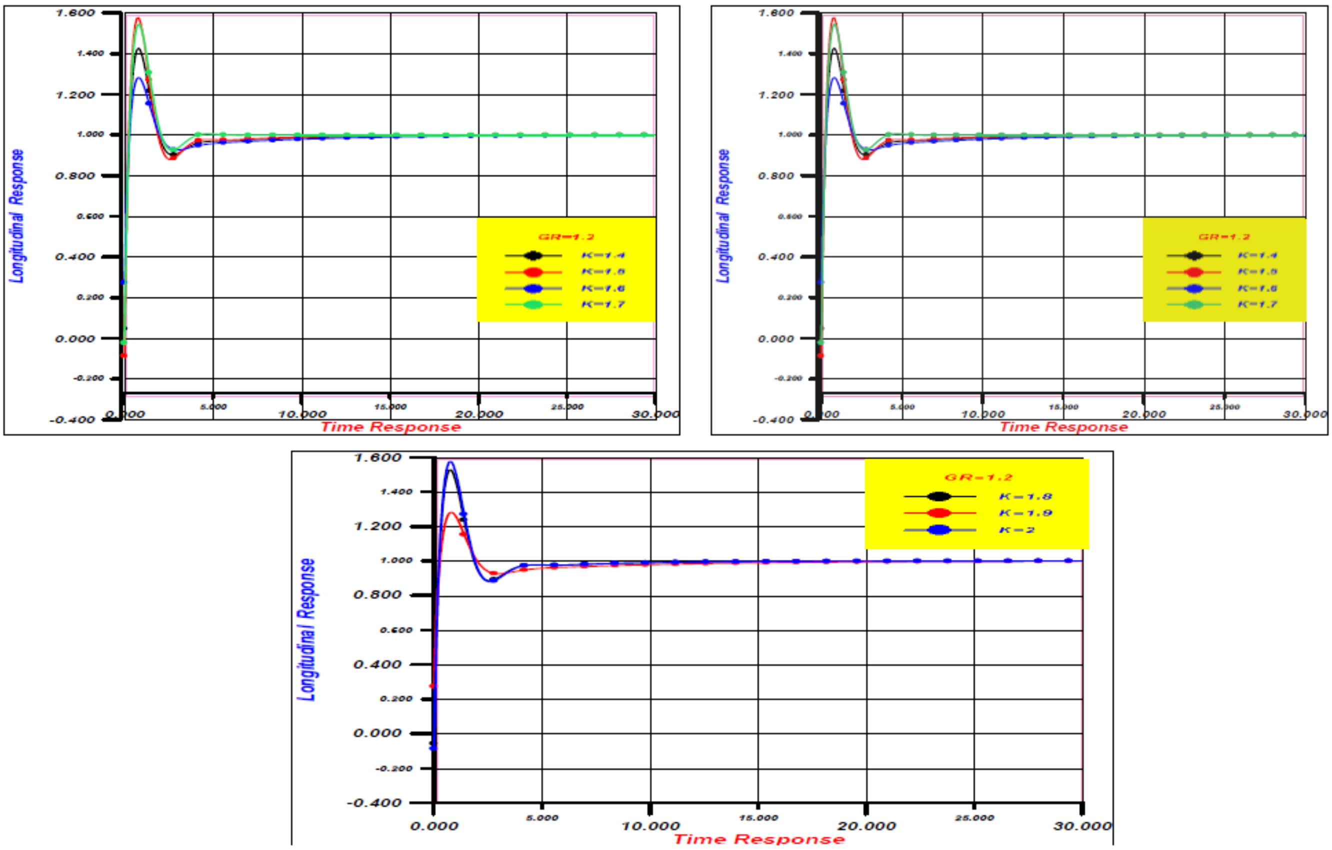


Figure (4) Longitudinal Response verses Time response for GR=1.3 and Different Gain Value

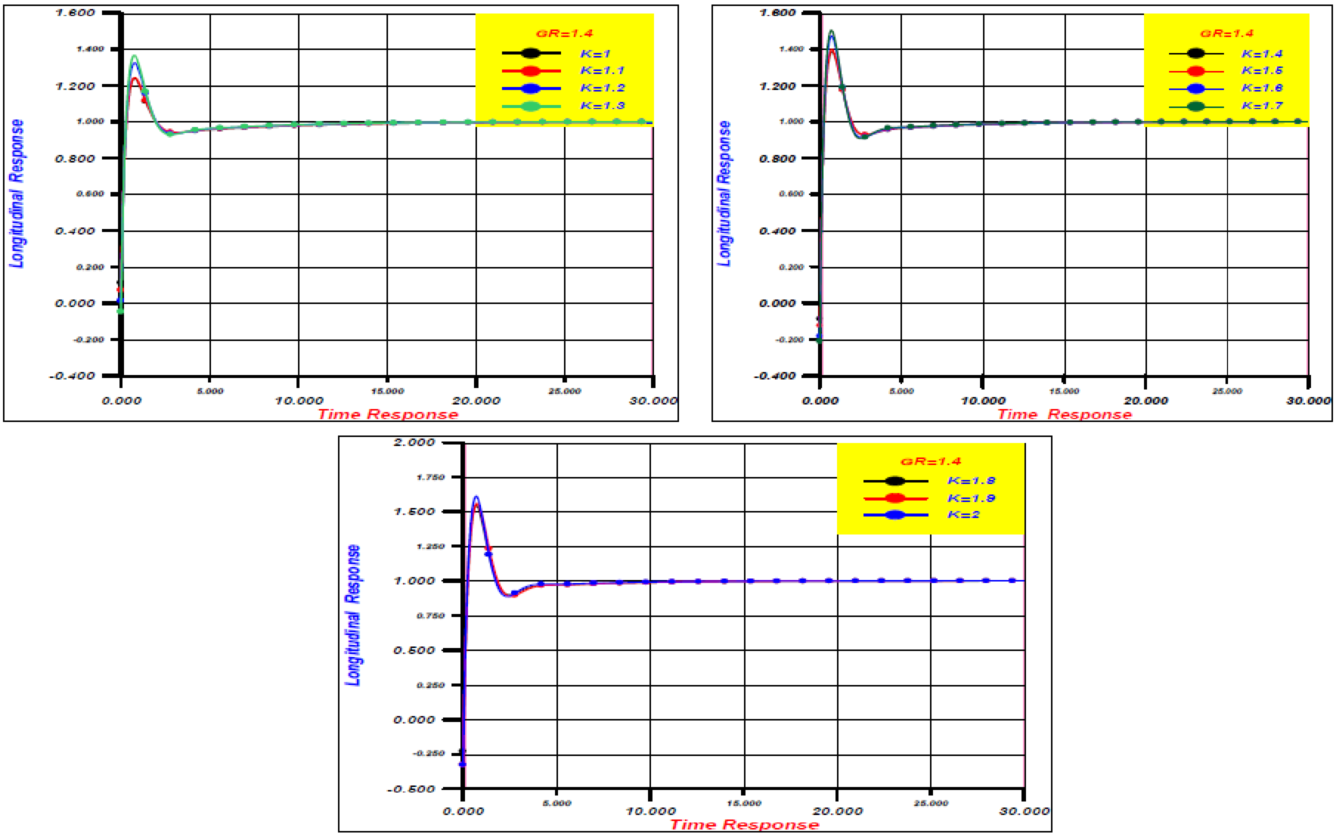


Figure (5) Longitudinal Response verses Time response for GR=1.4 and Different Gain Value

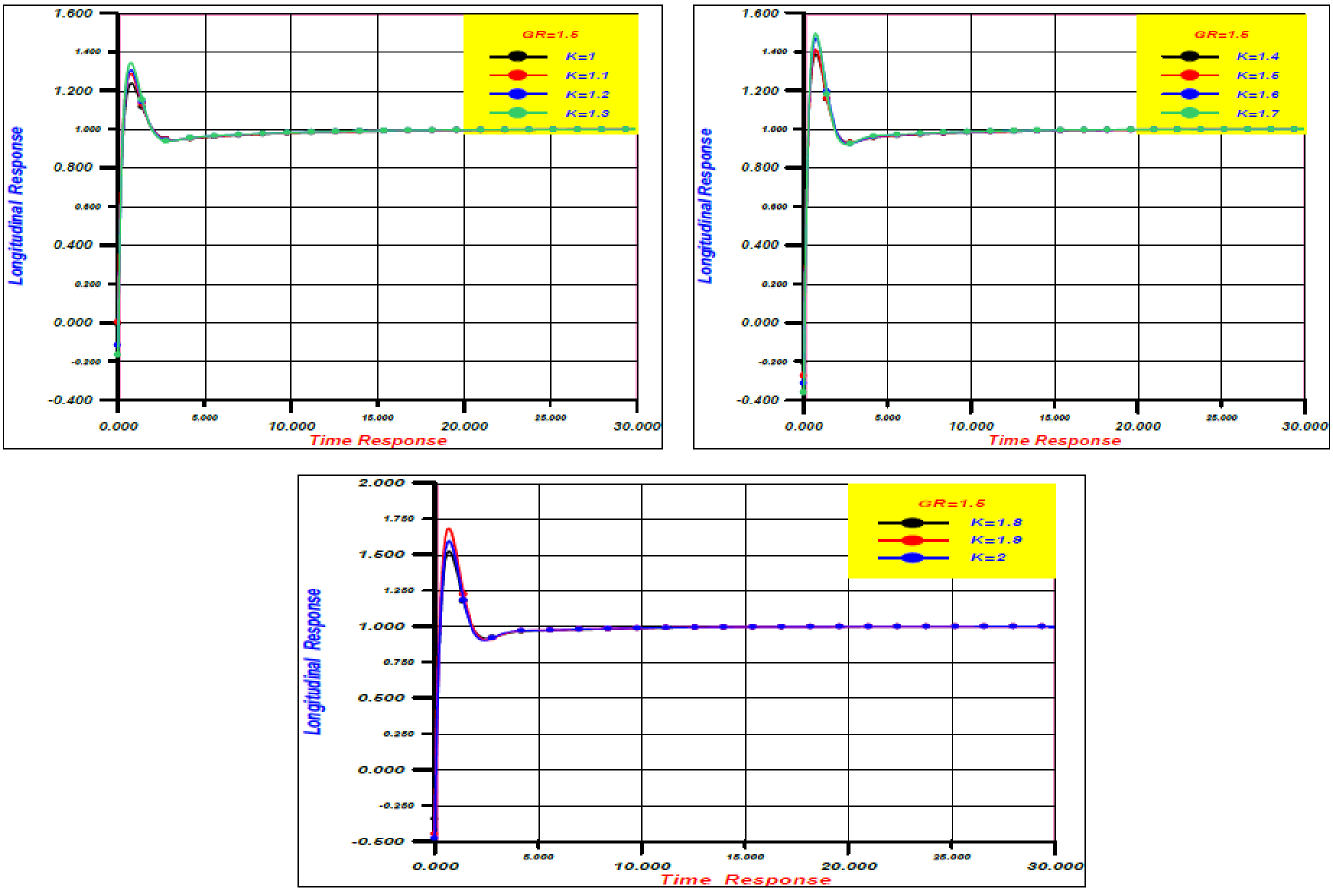


Figure (6) Longitudinal Response verses Time response for GR=1.5 and Different Gain Value

Published By:
Blue Eyes Intelligence Engineering
& Sciences Publication

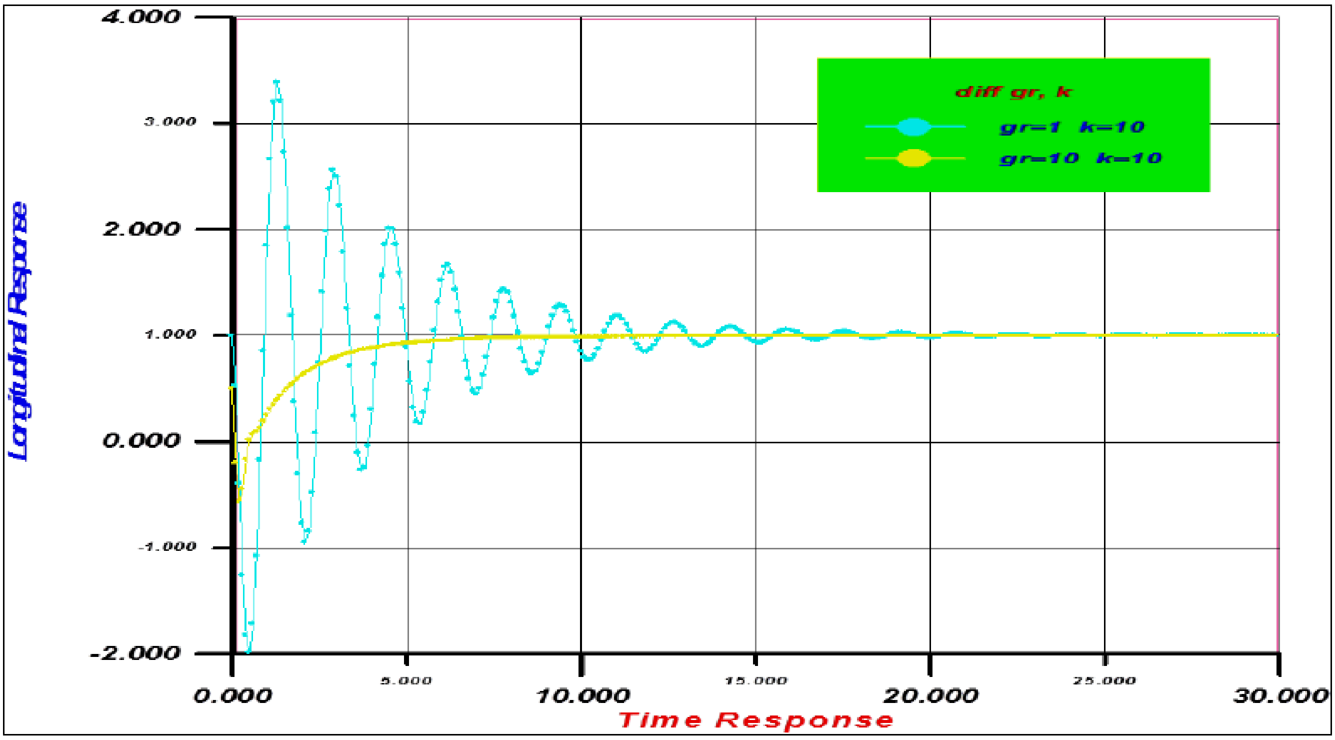
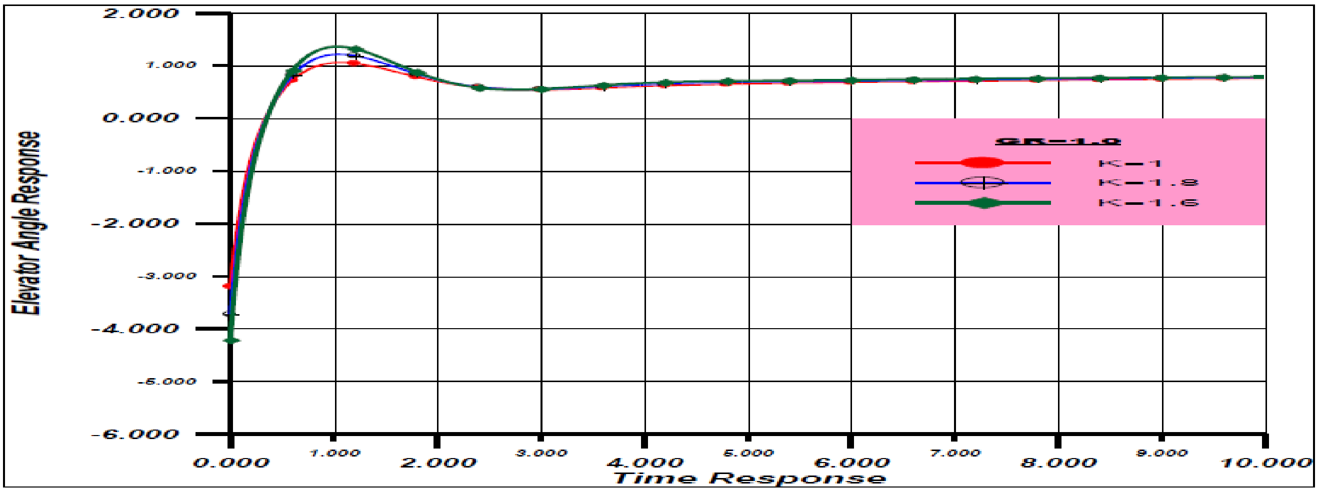
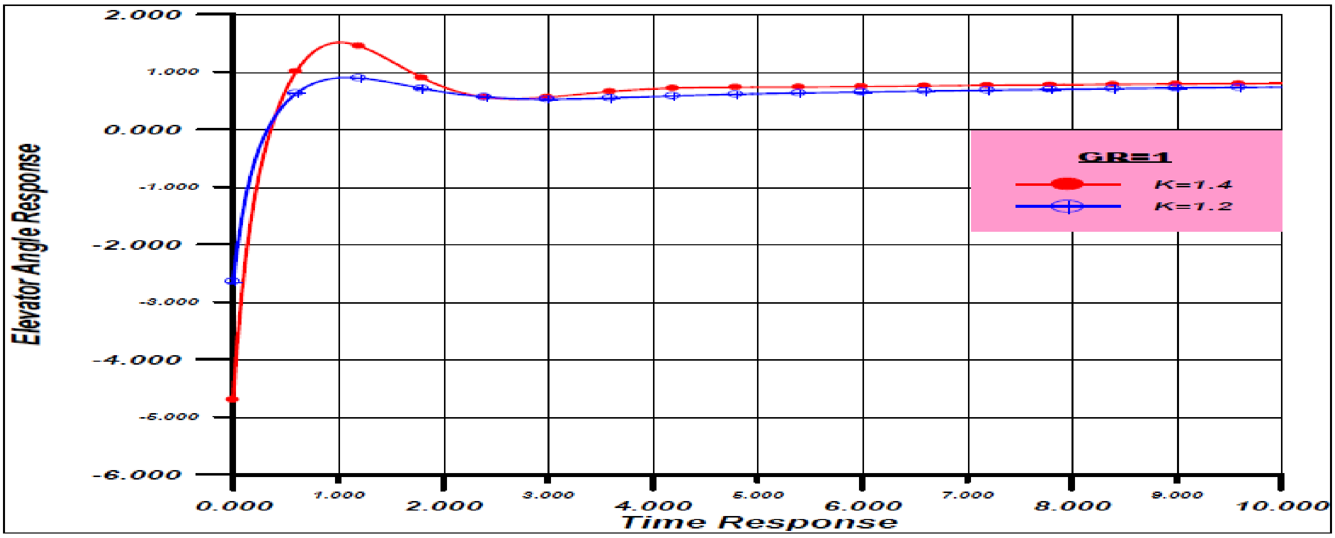


Figure (7) Longitudinal Response verses Time response for Different GR and Different Gain Value





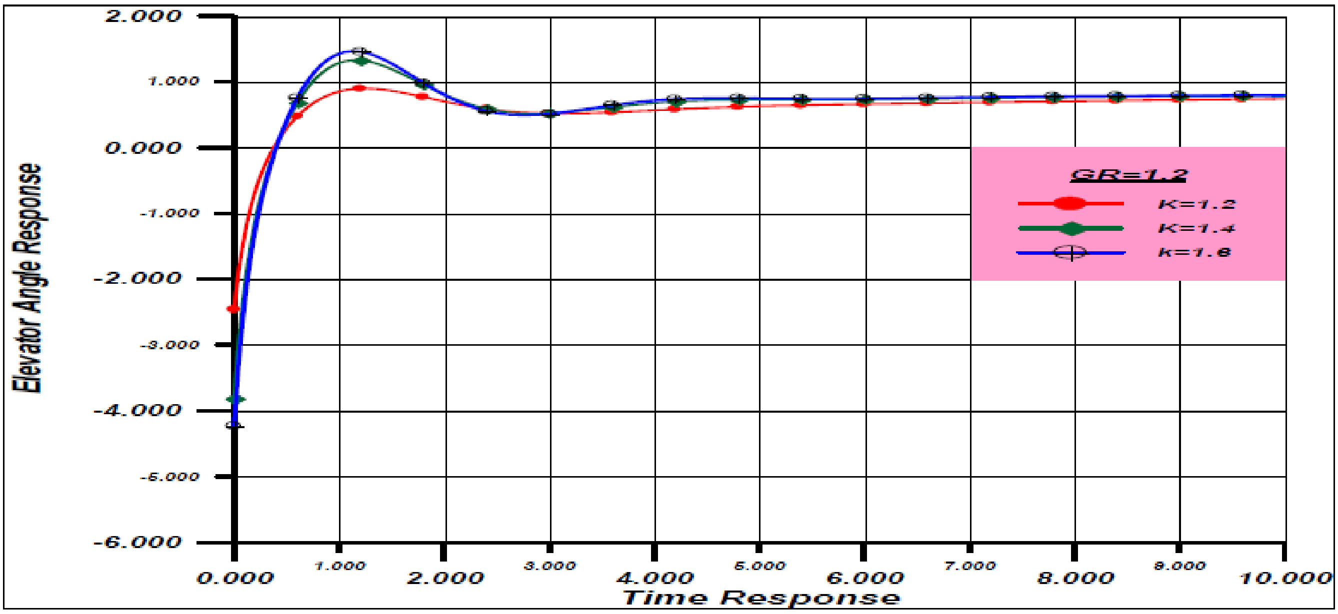


Figure (8) Elevator Angle Response for GR=1 and different "k"

3.2. Discussion

The purpose of this design to obtain largest gain attainable required and the smallest peak overshoot and shortest time setting .From table (3) and figures (8) GR=1.19, maximum K is equal 1.4 and for more than that the system is not stable.

It can be observe from table (1) that by increasing K at constant value of GR, the real part of complex root will decrease and the imaginary part will increase, this mean that the overshoot will increase too and setting time will also increase, so it must have a choose between the largest gain attainable, shortest peak overshoot and shortest time setting. Also by increasing at constant K the real part of complex root will increase and imaginary part will increase too and that increase the peak overshoot and better stability.

So it can't go beyond GR=1.19 and K=1.4 because it is limited by aileron deflection and case study design data which was equal to 5 rad. After compromising all the values of K and GR searching for our design requirement which given above, it found that GR=1.19, K=1.4 best selection which must be check it with actuator slewing time (time of actuator to deflect), it can be notice that the maximum value of actuator slewing rate at GR=1.19 and k=1.4 was equal to 43.668 rad /sec and the design value of actuator slewing rate is equal to 90 rad/sec, that mean the selection value with limit. Also it can be observe that as K increasing the actuator slewing time will also increase and for increasing GR at constant K the actuator slewing time remain same.

IV. CONCLUSIONS

It can conclude that the response of a control system is examined by root locus method and it depend on real root of the characteristic equation of close loop transfer function and this root depend on K and GR so by increasing K the root shift to right of root locus which decrease the stability but by increasing GR the root shift to the left which increases the stability also increasing K and GR depend on max aileron deflection. The requirement of maximum actuator slewing rate depends on K only because by increasing GR at constant K the maximum actuator slewing rate will remain the same.

REFERENCES

1. Etkin "Dynamics of Flight Stability and Control "John Willey, 2002. http:\\www.Amazon Online Reader Dynamics of Flight Stability and Control.mht
2. P.garnell and D.J. East "Guided weapon Control System" Royal military college of science, England, 2001. www.amazon.com/Guided-Weapon-Control-Systems-Garnell/dp/0080254683 - 204k -
3. John H. Block lock "Automatic Control of Aircraft and Missiles" John Willey and sonic, 2006. ttp:\\www.Amazon_com Automatic Control of Aircraft and Missiles John H_ Blake lock Books.mht
4. Alan Pope "Aerodynamic Of Supersonic Flight Optimum Aeronautical Publications"2002.n http:\\www .Amazon_com Used and New AERODYNAMICS OF SUPERSONIC FLIGHT an Introduction.mht
5. J.J D'Azzo. C.H.Houpis "Feedback Control System Analysis and Synthesis" Mc Graw. Hill, .2006. http:\\www.Customer Image Gallery for Feedback Control System Analysis and Synthesis.mht 7. John J.D'Azzo C.H. Houpis "linear Control System Analysis and Design"
6. Gustav Doetsch. "Guide to the Applications of the Laplace and B-Transformation ", 2008. http:\\www.Amazon_com Gustav Doetsch Books.mht
7. J.J D'Azzo. C.H.Houpis "Linear Control System, Analysis and Design ", 2006.
8. Flight Dynamic NOTES C. I. T. U. k C. I. T. U.k, 1986.